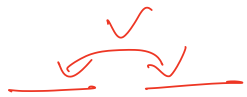
→ Reminder

(1) Linear deg. dimer



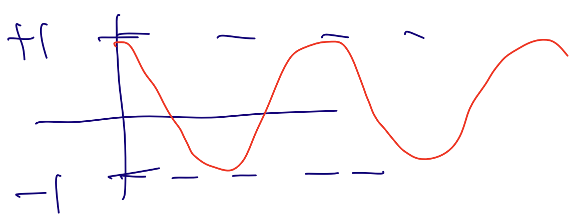
$$i\dot{\phi}_1 = V\phi_2, \qquad i\dot{\phi}_2 = V\phi_1.$$

$$\rightarrow \quad \rho = \begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix} \rightarrow \left. \begin{aligned} i\dot{\phi}_1\phi_1^* &= V\phi_2\phi_1^* \\ i\dot{\phi}_1^*\phi_1 &= -V\phi_2^*\phi_1 \end{aligned} \right\} \Rightarrow \dot{\rho}_{11} = \dots$$

$$\left. \begin{aligned} i\dot{\rho}_{11} &= -V(\rho_{12}-\rho_{21}) \\ i\dot{\rho}_{22} &= V(\rho_{12}-\rho_{21}) \\ i\dot{\rho}_{12} &= -V(\rho_{11}-\rho_{22}) \\ i\dot{\rho}_{21} &= V(\rho_{11}-\rho_{22}) \end{aligned} \right\} \Rightarrow \begin{aligned} i\dot{P} &= -2V\mathcal{Q} \\ i\dot{\mathcal{Q}} &= -2VP \end{aligned} \quad \Bigg| \quad \begin{aligned} p &= P \\ q &= i\mathcal{Q} \end{aligned}$$

$$\left. \begin{aligned} \dot{p} &= 2Vq \\ \dot{q} &= -2Vp \end{aligned} \right\} \quad \ddot{p} = 2V(-2Vp), \qquad \boxed{\ddot{p} + (2V)^2 p = 0}$$

$$P_0 = 1 \quad \rightarrow \quad \boxed{P(t) = \cos(2Vt)}$$ prob diff

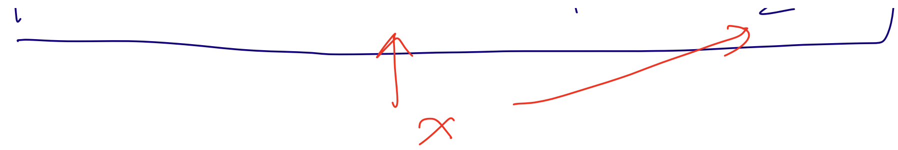


(2) Nonlinear deg. dimer

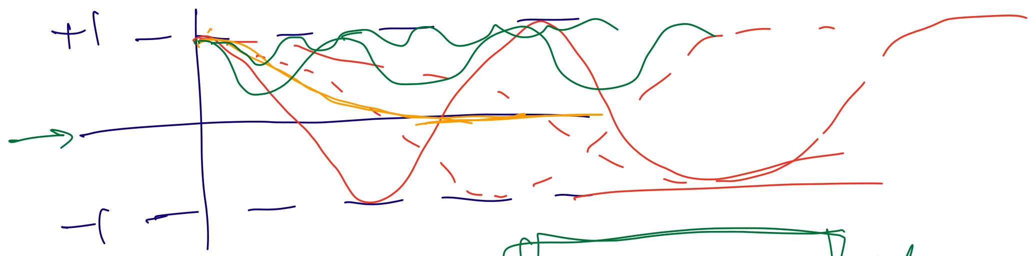
$$i\dot{\phi}_1 = V\phi_2 - \chi|\phi_1|^2\phi_1$$
$$i\dot{\phi}_2 = V\phi_1 - \chi|\phi_2|^2\phi_2.$$

p, q, r eqs →

p-equation: $$\boxed{\ddot{p} + \left[(2V)^2 + 2\chi V\left(r_0 - \frac{\chi}{4V}p_0\right)\right]p - \frac{\chi^2}{2}p^3 = 0}$$

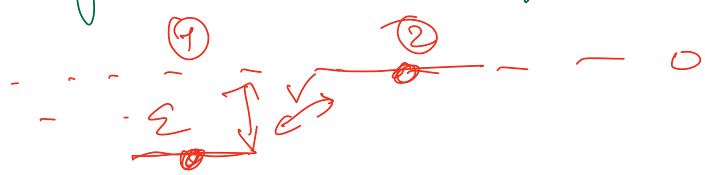


If $x=0$ $\quad \ddot{p} + (2V)^2 p = 0$



Nonlinearity $\boxed{\text{slows down}}$ the motion until we have self trapping & then incomplete oscillation.

③ TET



$$\boxed{\begin{aligned} i\dot{\psi}_1 &= -\varepsilon\psi_1 + V\psi_2 - \chi_1|\psi_1|^2\psi_1 \\ i\dot{\psi}_2 &= 0 \quad\; + V\psi_1 - \chi_2|\psi_2|^2\psi_2 . \end{aligned}}$$

$$i\dot{\psi}_1\psi_1^* = \cancel{-\varepsilon\psi_1\psi_1^*} + V\psi_2\psi_1^* - \chi_1|\psi_1|^2\psi_1\psi_1^*$$

$$i\dot{\psi}_1^*\psi_1 = \cancel{\varepsilon\psi_1^*\psi_1} - V\psi_2^*\psi_1 + \chi_1|\psi_1|^2\psi_1^*\psi_1$$

$$\boxed{i\dot{\rho}_{11} = -V(\rho_{12} - \rho_{21})} \quad (\mathrm{I})$$

$$i\dot{\psi}_2\psi_2^* = \underline{V\psi_1\psi_2^*} - \chi_2|\psi_2|^2\psi_2\psi_2^*$$

$$i\,\dot{\varphi}_2^*\varphi_2 = -V\varphi_1^*\varphi_2 + \varkappa_2|\varphi_2|^2\varphi_2^*\varphi_2$$

$$\boxed{i\dot{\rho}_{22} = V(\rho_{12}-\rho_{21})} \quad (\mathrm{II})$$

$$i\,\dot{\varphi}_1\varphi_2^* = -\epsilon\varphi_1\varphi_2^* + V\varphi_2\varphi_2^* - \varkappa_1|\varphi_1|^2\varphi_1\varphi_2^*$$

ρ_{11} $-\rho_{12}$

$$i\,\dot{\varphi}_2^*\varphi_1 = -V\varphi_1^*\varphi_1 + \varkappa_2|\varphi_2|^2\varphi_2^*\varphi_1$$

ρ_{22} $-\rho_{12}$

$$\boxed{i\dot{\rho}_{12} = -\epsilon\rho_{12} - V(\rho_{11}-\rho_{22}) - \rho_{12}(\varkappa_1\rho_{11}-\varkappa_2\rho_{22})} \quad (\mathrm{III})$$

$$i\,\dot{\varphi}_1^*\varphi_2 = \epsilon\varphi_1^*\varphi_2 - V\varphi_2^*\varphi_2 + \varkappa_1|\varphi_1|^2\varphi_1^*\varphi_2$$

ρ_{21}

$$i\,\dot{\varphi}_2\varphi_1^* = V\varphi_1\varphi_1^* - \varkappa_2|\varphi_2|^2\varphi_2\varphi_1^*$$

ρ_{21} (IV)

$$\boxed{i\dot{\rho}_{21} = \epsilon\rho_{21} + V(\rho_{11}-\rho_{22}) + \rho_{21}(\varkappa_1\rho_{11}-\varkappa_2\rho_{22})}$$

TET ~~EQUATIONS~~ EQUATIONS FOR DENSITY MATRIX

$i\dot{\rho}_{11} = -V(\rho_{12}-\rho_{21})$	$i\dot{\rho}_{12} = -\epsilon\rho_{12} - V(\rho_{11}-\rho_{22}) - \rho_{12}(\varkappa_1\rho_{11}-\varkappa_2\rho_{22})$
$i\dot{\rho}_{22} = V(\rho_{12}-\rho_{21})$	$i\dot{\rho}_{21} = \epsilon\rho_{21} + V(\rho_{11}-\rho_{22}) + \rho_{21}(\varkappa_1\rho_{11}-\varkappa_2\rho_{22})$

Notice

$$\varkappa_1\rho_{11}-\varkappa_2\rho_{22}$$

$$\begin{aligned} x_1 &= -x \\ x_2 &= +x \end{aligned}$$

$(x > 0)$

$x = -x_1 = x_2$

$$x_1 \rho_{11} - x_2 \rho_{22} = -x\rho_{11} - x\rho_{22} = -x\underbrace{(\rho_{11}+\rho_{22})}_{1} = -x$$

$$\boxed{\begin{array}{l|l} i\dot\rho_{11} = -V(\rho_{12}-\rho_{21}) & i\dot\rho_{12} = -\epsilon\rho_{12} - V(\rho_{11}-\rho_{22}) + x\rho_{12} \\ i\dot\rho_{22} = V(\rho_{12}-\rho_{21}) & i\dot\rho_{21} = \epsilon\rho_{21} + V(\rho_{11}-\rho_{22}) - x\rho_{12} \end{array}}$$

$$(\varepsilon, x_1, x_2) \longrightarrow (\varepsilon, x) \qquad \boxed{x_2 = -x_1 = x}$$

$$i(\underbrace{\dot\rho_{11}-\dot\rho_{22}}_{P}) = -2V(\overbrace{\rho_{12}-\rho_{21}}^{Q}) \longrightarrow \boxed{i\dot P = -2VQ} \quad (\alpha)$$

$$i(\dot\rho_{12}-\dot\rho_{21}) = -\epsilon(\rho_{12}+\rho_{21}) - 2V(\rho_{11}-\rho_{22}) + x(\rho_{12}+\rho_{21})$$

$$\boxed{i\dot Q = -(\epsilon - x)R - 2VP} \quad (\beta)$$

$$i(\dot\rho_{12}+\dot\rho_{21}) = -\epsilon(\rho_{12}-\rho_{21}) + x(\rho_{12}-\rho_{21})$$

$$\boxed{i\dot R = -(\epsilon - x)(\rho_{12}-\rho_{21})} \quad (\gamma)$$

$P = \rho_{11} - \rho_{22}$

$Q = \rho_{12} - \rho_{21}$

$R = \rho_{12} + \rho_{21}$

$$\left.\begin{aligned} p &= P \\ q &= iQ \\ r &= R \end{aligned}\right\}$$

$(\alpha) \to \boxed{\dot p = 2Vq}$ (α')

(β) $\boxed{\dot q = -(\epsilon - x) r - 2Vp}$ (β')

$$\boxed{\varepsilon' = \varepsilon - x}$$

$$\boxed{\dot{r} = (\epsilon - x) q} \quad (\gamma)$$

$$\dot{p} = 2\nu q \quad (A)$$

$$\dot{q} = -\varepsilon' r - 2\nu p \quad (B)$$

$$\dot{r} = \varepsilon' q \quad (C)$$

$$\dot{r} = \varepsilon' \frac{\dot{p}}{2\nu} = \frac{\varepsilon'}{2\nu} \dot{p}$$

$$r - r_0 = \frac{\varepsilon'}{2\nu} (p - p_0)$$

$$(C') \quad \boxed{r = \left(r_0 - \frac{\varepsilon'}{2\nu} p_0\right) + \frac{\varepsilon'}{2\nu} p}$$

$$(B) \xrightarrow{(A)(C')} \quad \frac{\ddot{p}}{2\nu} = -\varepsilon' \left[\left(r_0 - \frac{\varepsilon'}{2\nu} p_0\right) + \frac{\varepsilon'}{2\nu} p\right] - 2\nu p$$

$$\ddot{p} = -2\nu\varepsilon' \left(r_0 - \frac{\varepsilon'}{2\nu} p_0\right) - \varepsilon'^2 p - (2\nu)^2 p$$

$$\boxed{\ddot{p} + \left[(2\nu)^2 + \varepsilon'^2\right] p = -2\nu\varepsilon' \left(r_0' - \frac{\varepsilon'}{2\nu} p_0\right)}$$

Linear !

Init cond $r_0 = 0$, $p_0 = 1$

$$\boxed{\ddot{p} + \left[\underbrace{(2\nu)^2 + \epsilon'^2}_{\Omega^2}\right] p = \varepsilon'^2} \qquad \boxed{\varepsilon' = \varepsilon - x}$$

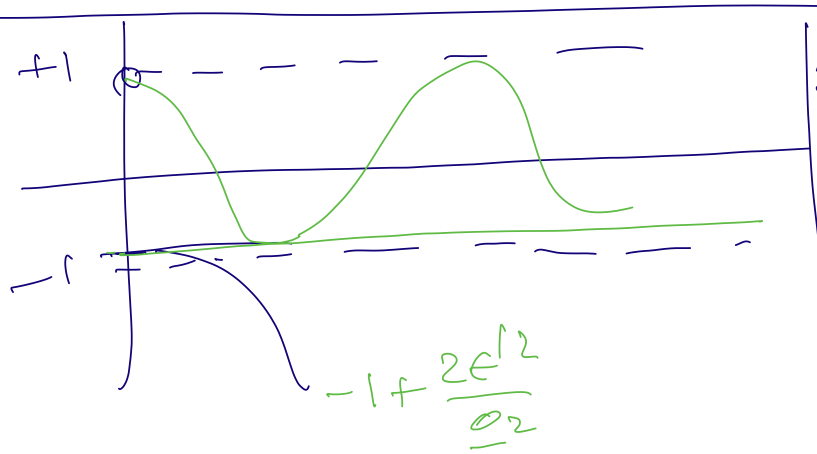
$$\Omega = \sqrt{(2\nu)^2 + \epsilon'^2}$$

$$\boxed{p(t) = A \cos \Omega t + B \sin \Omega t + \frac{\varepsilon'^2}{\Omega^2}}$$

$$p(0) = A + \frac{\epsilon'^2}{\Omega^2} = 1 \quad \rightarrow \quad A = 1 - \frac{\varepsilon'^2}{\Omega^2}$$

$\dot{P}(0) = B = 0$

$$\boxed{P(t) = \left(1 - \frac{\epsilon'^2}{\Omega^2}\right)\cos\Omega t + \frac{\epsilon'^2}{\Omega^2}}$$



$\Omega t = 0$

$P(0) = 1 - \frac{\epsilon'^2}{\Omega^2} + \frac{\epsilon'^2}{\Omega^2} = 1$

$\Omega t = \frac{\pi}{2}$ $\quad \cos\Omega t = 0$

$P(\frac{\pi}{2}) = \frac{\epsilon'^2}{\Omega^2}$

$\Omega t = \pi$ $\quad -\cos\Omega t = -1$

$P(\pi) = -1 + \frac{2\epsilon'^2}{\Omega^2}$

$\Omega^2 = (2V)^2 + \epsilon'^2$

$$\boxed{\epsilon'^2 = \epsilon - x}$$

special value $\epsilon' = 0$ $\rightarrow$ $\boxed{\epsilon = x}$

$$\boxed{P(t) = \cos 2Vt}$$ $\leftarrow$ linear dimer result

$$\boxed{\epsilon = -x_1 = x_2}$$

Fully resonant condition

& TET

[illegible] TET is very selective!