University of Crete, School of Sciences and Engineering, Department of Physics, Heraklion

ΦΥΣ 572: Physics of Semiconductor Devices

Lecture 7a
Introduction to Bipolar Junction Transistor (BJT) and Ideal BJT analysis

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Alexandros I. Georgakilas

Professor, Section of Applied Physics
Micro/Nano-electronics Research Group

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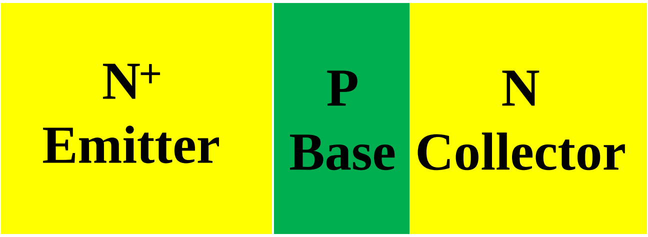
Bipolar Junction Transistor

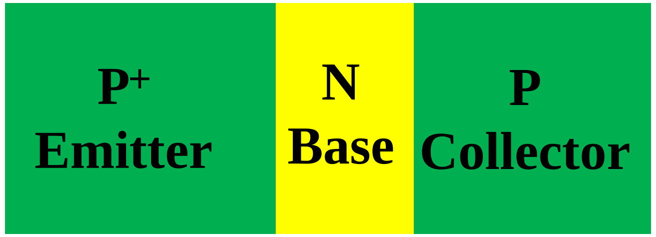
A Bipolar Junction Transistor (BJT) consists of three regions with alternating doping and conductivity type: NPN or PNP

- **The three regions are called Emitter, Base and Collector and correspond to three terminals / contacts of the BJT device**
- **Two neighbouring pn junctions interact through their common central region (Base), which is very narrow compared to the minority carrier diffusion length within it ($W << L_{p(n)}$)**
- **Bipolar device means that its operation (currents) is based on both electrons and holes**
- **A BJT can be used for current and voltage gain (amplification) or as a switch (ON and OFF states)**

NPN BJT



PNP BJT



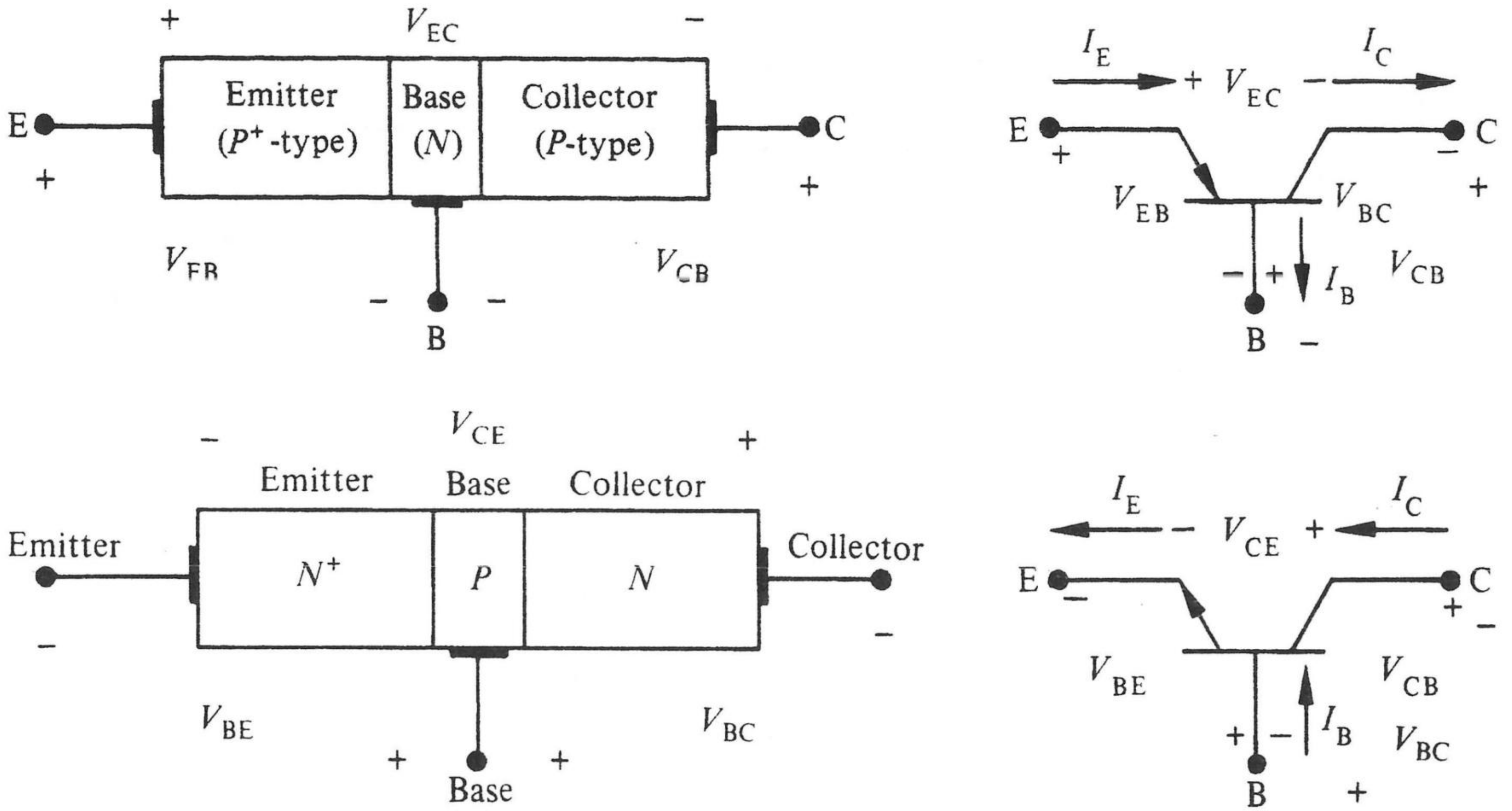
In principle, a BJT could be a symmetrical device concerning the doping concentrations. However, the target of good device performance practically imposes the use of heavily doped Emitter

Definitions of currents and voltages

BJT physical device **BJT circuit symbol**

PNP BJT →

NPN BJT →

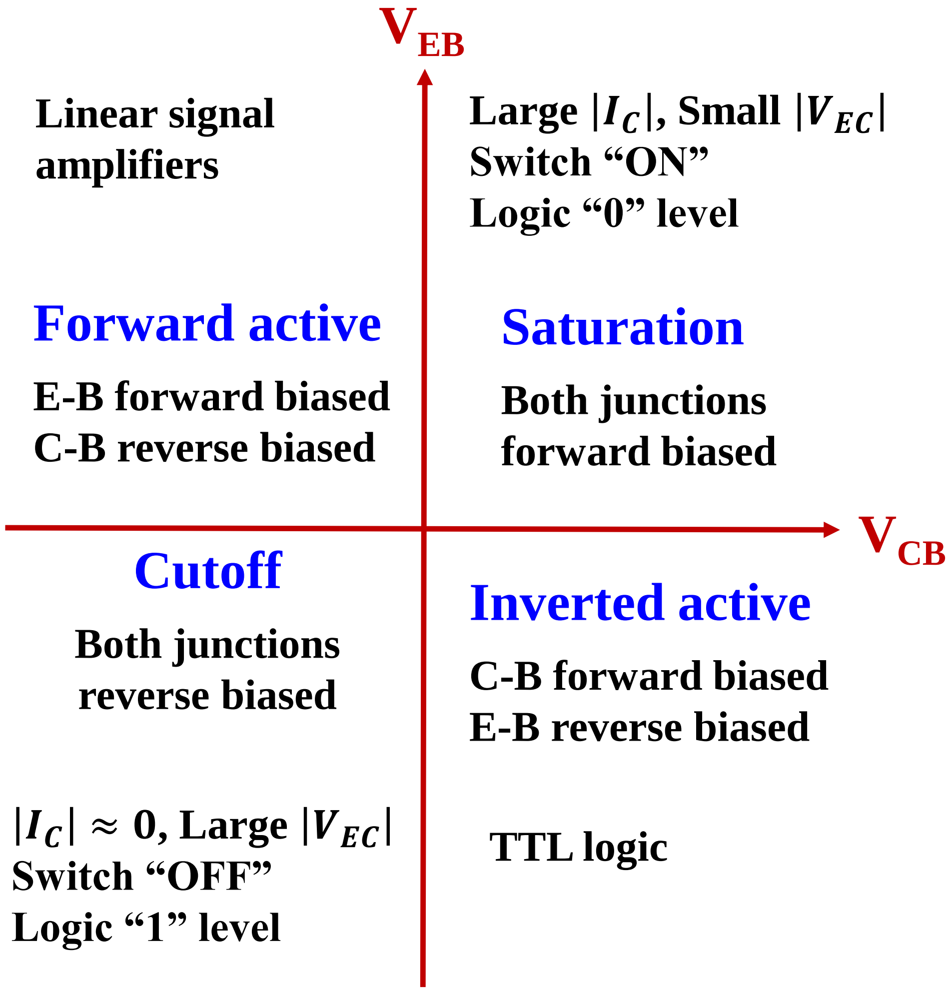


Figures from "The Bipolar Junction Transistor", G. W. Neudeck (Addison-Wesley)

Currents were defined to be positive when the device is biased to operate at the most usual condition

Four regions of BJT bias / operation – PNP case

PNP BJT



Linear signal amplifiers

Forward active

E-B forward biased
C-B reverse biased

Large $|I_C|$, Small $|V_{EC}|$
Switch “ON”
Logic “0” level

Saturation

Both junctions forward biased

Cutoff

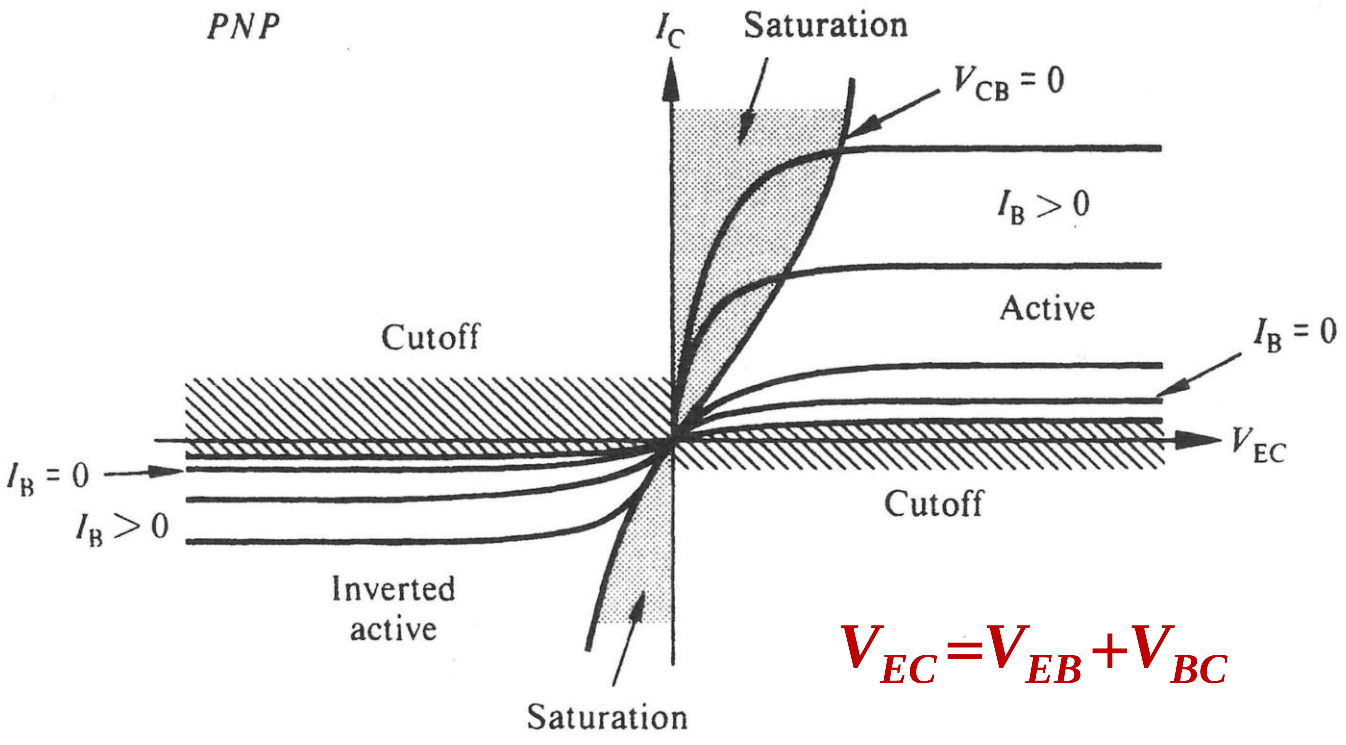
Both junctions reverse biased

$|I_C| \approx 0$, Large $|V_{EC}|$
Switch “OFF”
Logic “1” level

Inverted active

C-B forward biased
E-B reverse biased

TTL logic

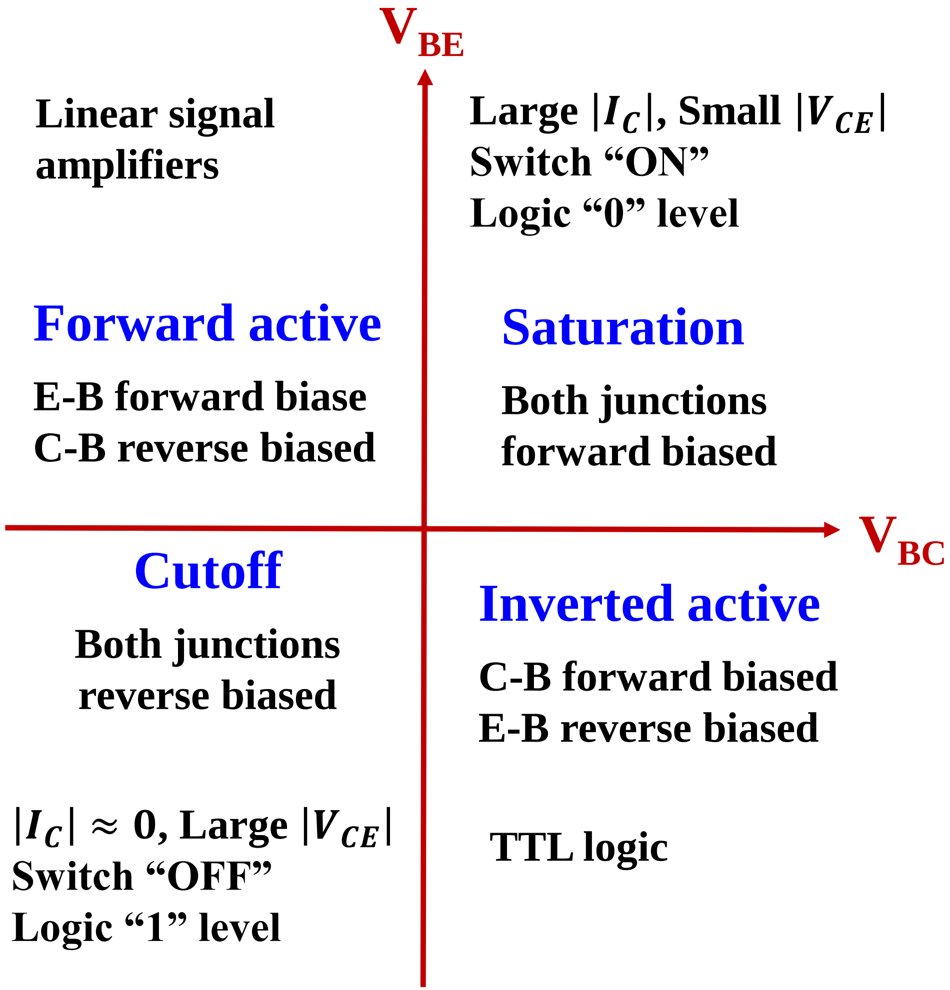


$$V_{EC} = V_{EB} + V_{BC}$$

From "The Bipolar Junction Transistor", G. W. Neudeck (Addison-Wesley)

Four regions of BJT bias / operation – NPN case

NPN BJT



V_{BE}

V_{BC}

Linear signal amplifiers

Forward active

E-B forward biase
C-B reverse biased

Large $|I_C|$, Small $|V_{CE}|$
Switch “ON”
Logic “0” level

Saturation

Both junctions forward biased

Cutoff

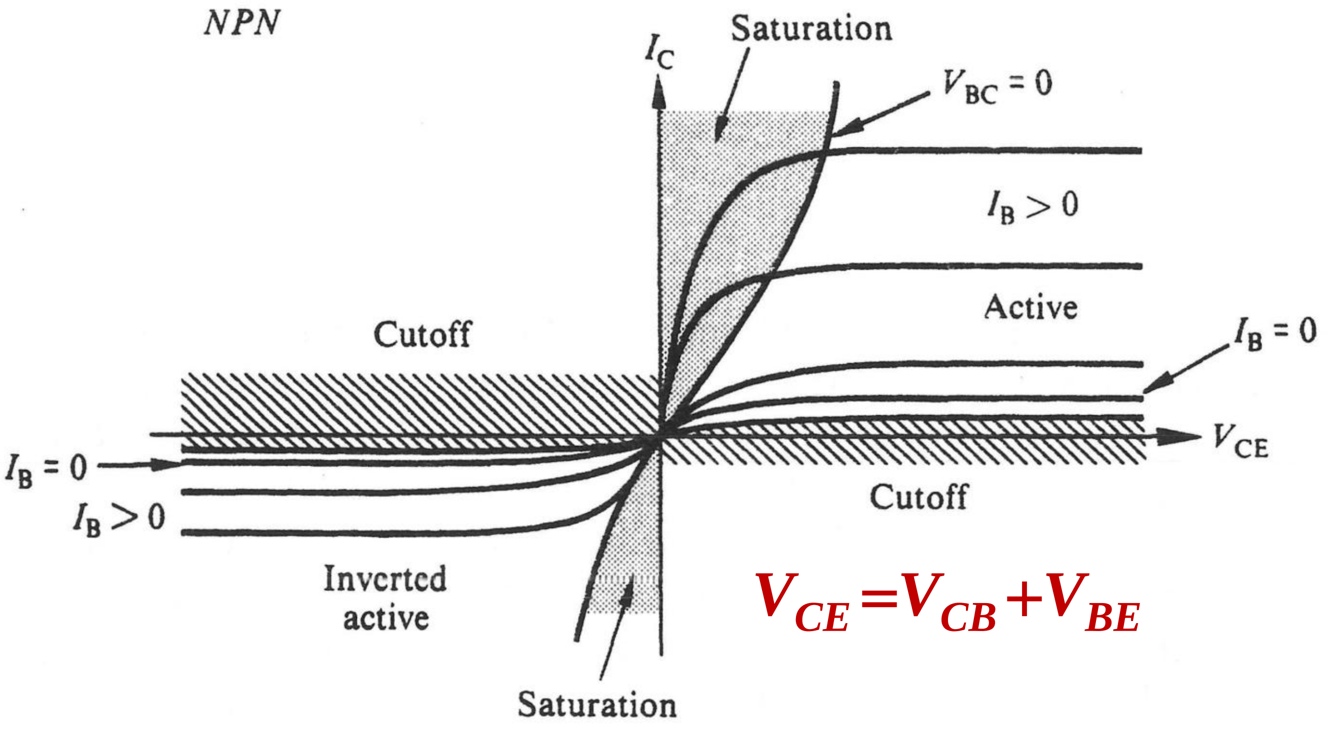
Both junctions reverse biased

$|I_C| \approx 0$, Large $|V_{CE}|$
Switch “OFF”
Logic “1” level

Inverted active

C-B forward biased
E-B reverse biased

TTL logic



$V_{CE} = V_{CB} + V_{BE}$

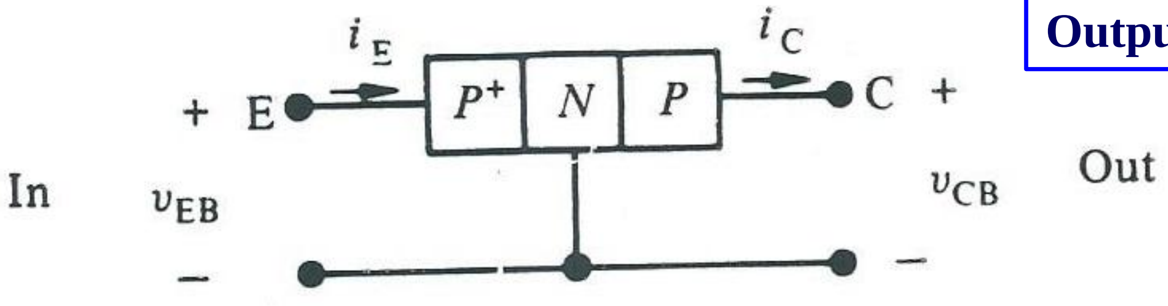
From "The Bipolar Junction Transistor", G. W. Neudeck (Addison-Wesley)

Three possible BJT connections in amplifier circuits

UoC Physics

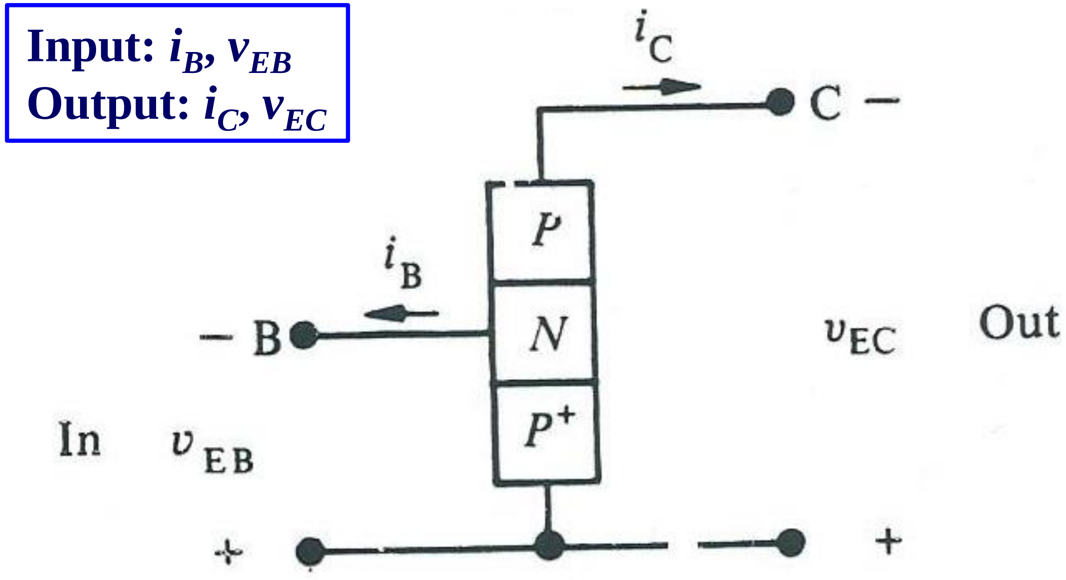
Common Base

Input: i_E, v_{EB}
Output: i_C, v_{CB}



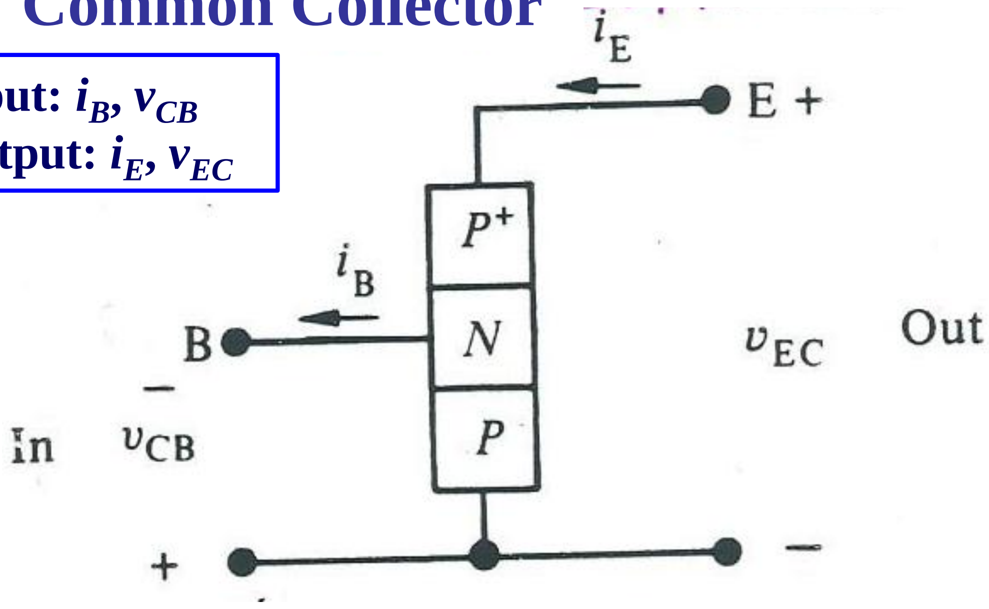
Common Emitter

Input: i_B, v_{EB}
Output: i_C, v_{EC}



Common Collector

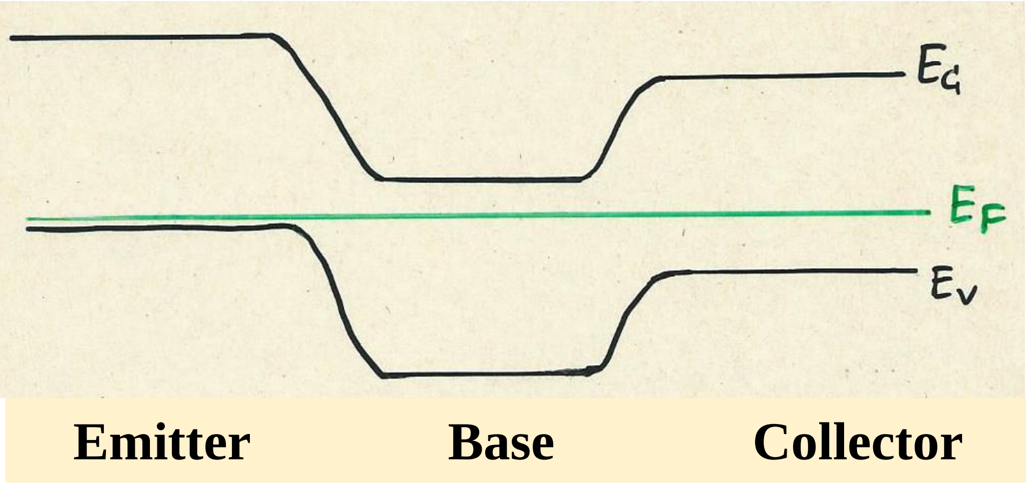
Input: i_B, v_{CB}
Output: i_E, v_{EC}



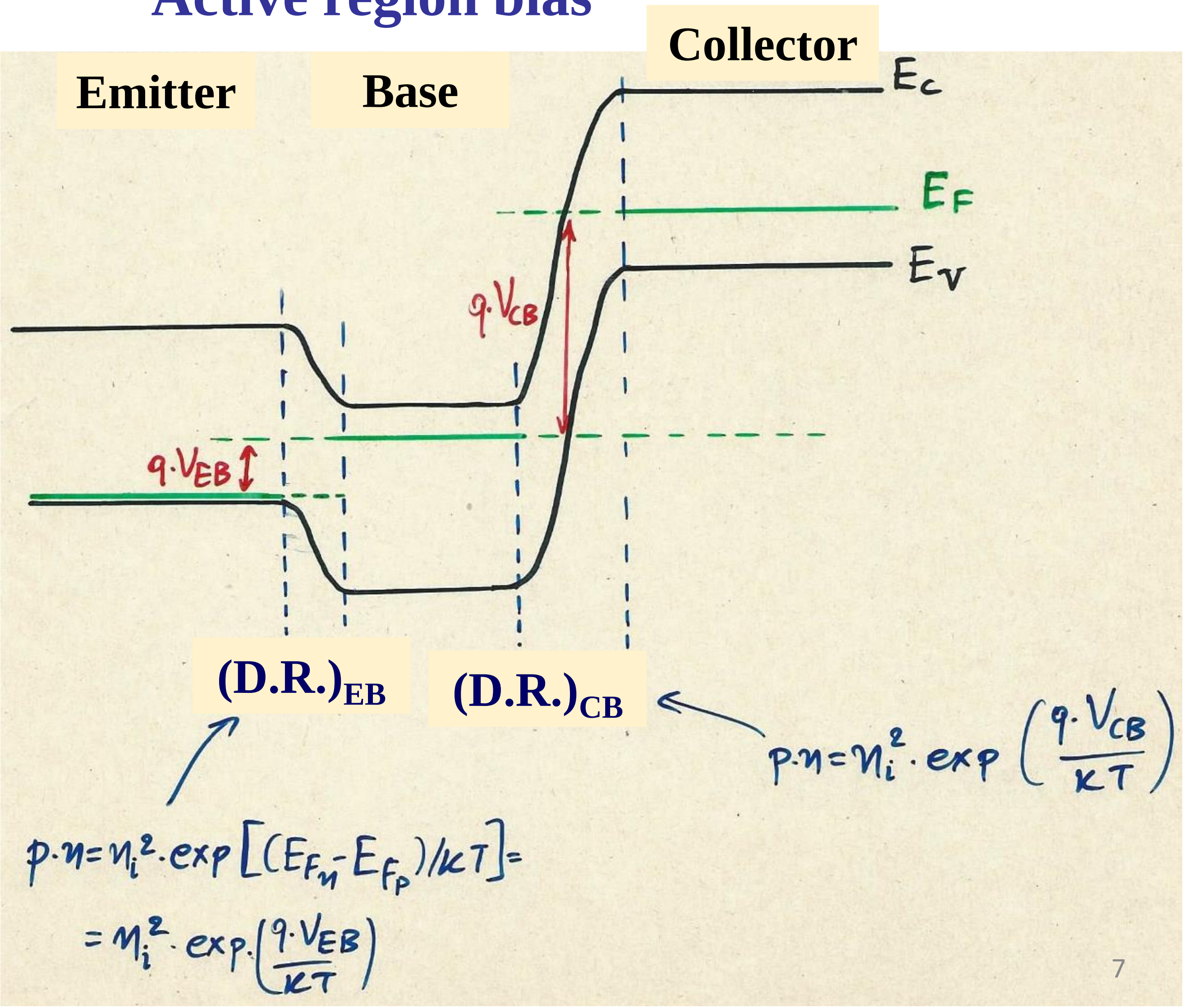
Figures from "The Bipolar Junction Transistor", G. W. Neudeck (Addison-Wesley)

Energy band diagrams for PNP BJT

Equilibrium



Active region bias

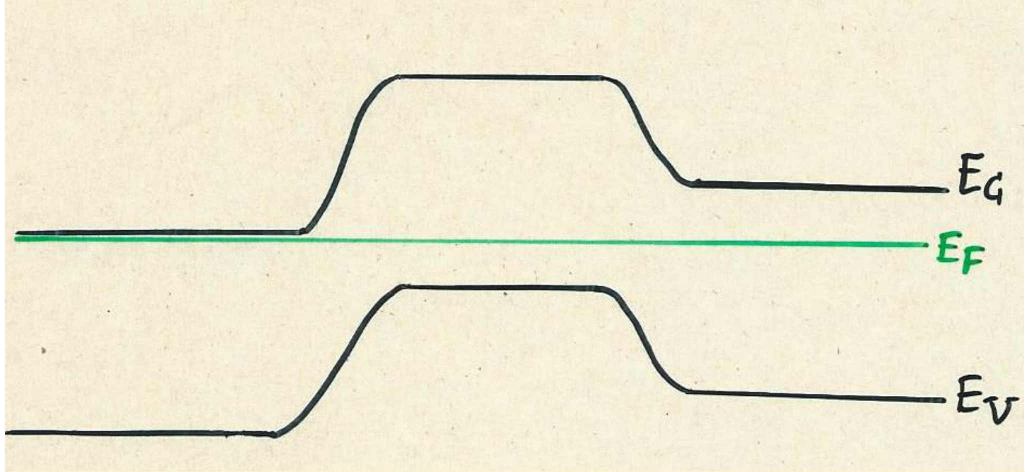


$$p \cdot n = n_i^2 \cdot \exp\left(\frac{q \cdot V_{CB}}{kT}\right)$$

$$p \cdot n = n_i^2 \cdot \exp[(E_{F_n} - E_{F_p})/kT] = $$
$$= n_i^2 \cdot \exp\left(\frac{q \cdot V_{EB}}{kT}\right)$$

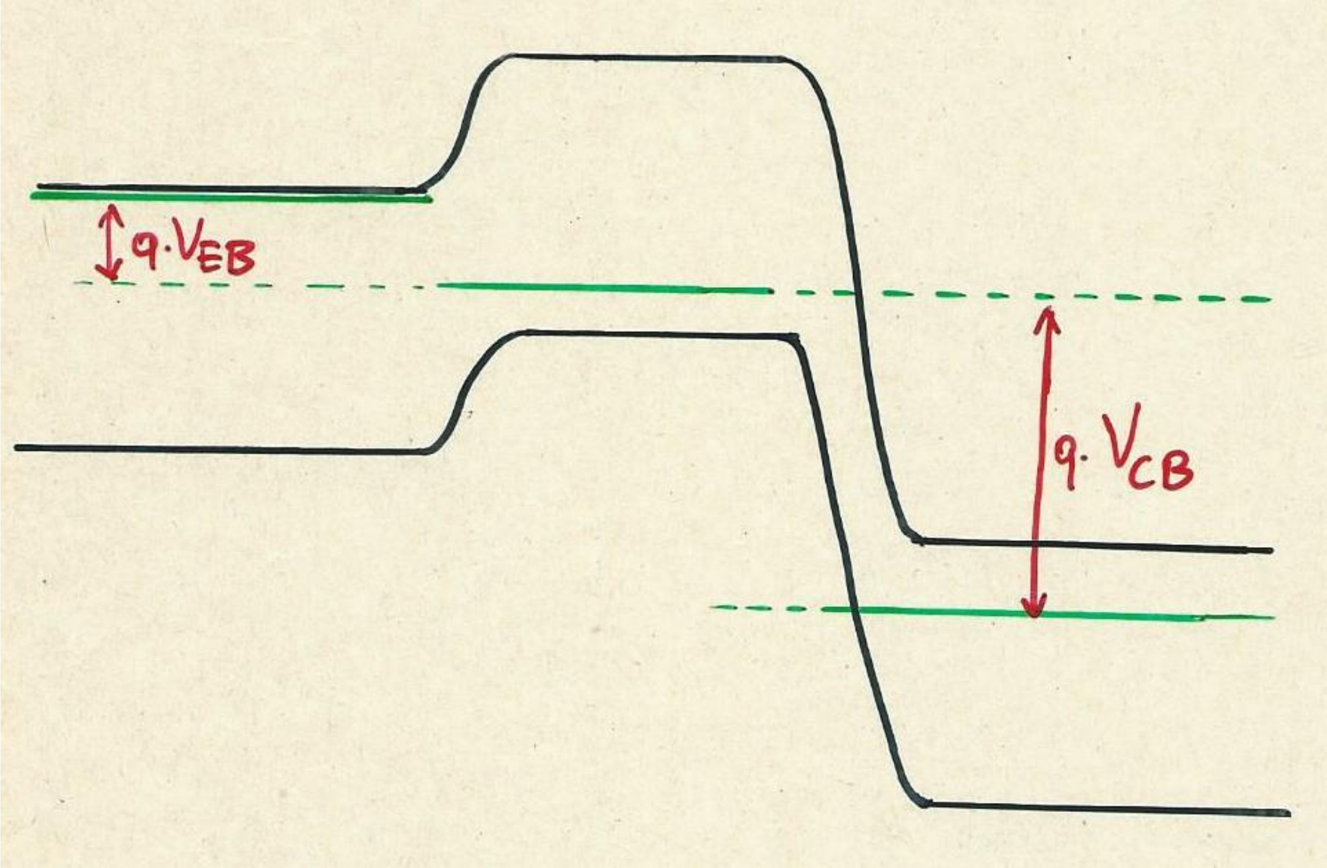
Energy band diagrams for NPN BJT

Equilibrium

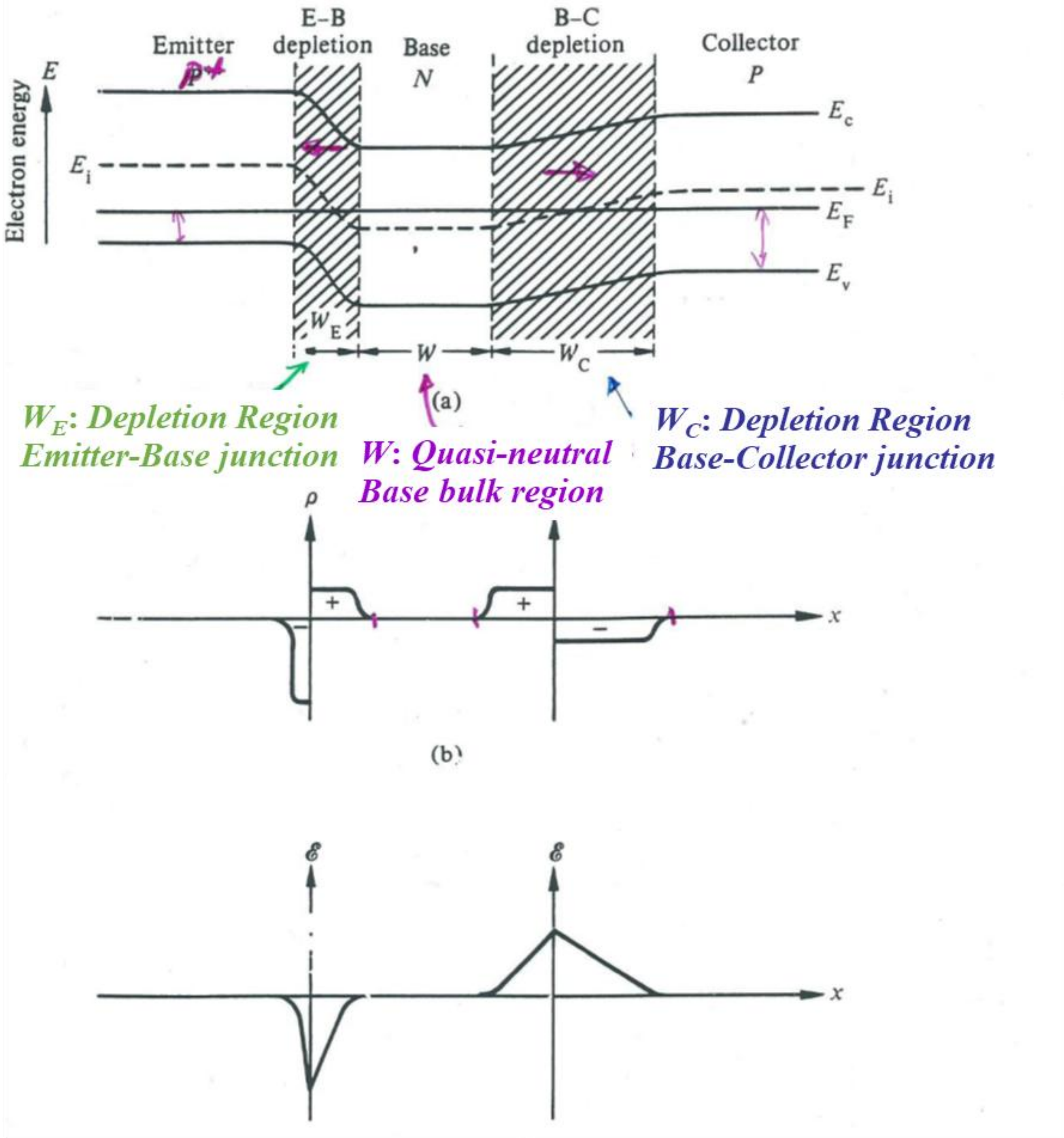


Emitter **Base** **Collector**

Active region bias



PNP at equilibrium – Charge and electric field



W_E: Depletion Region Emitter-Base junction

W: Quasi-neutral Base bulk region

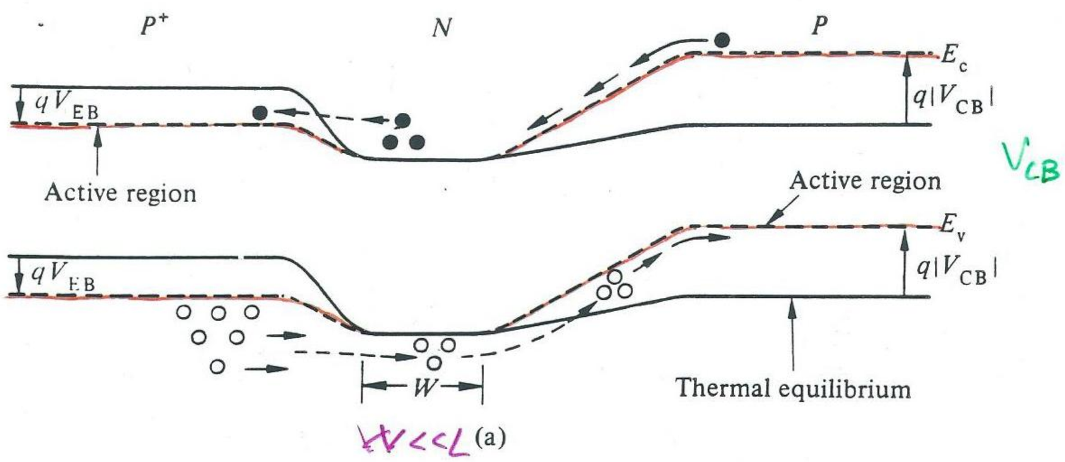
W_C: Depletion Region Base-Collector junction

A quasi-neutral Base bulk region of width W exists between the two depletion regions of widths W_E and W_C, formed at the Emitter-Base and Collector-Base pn junctions, respectively

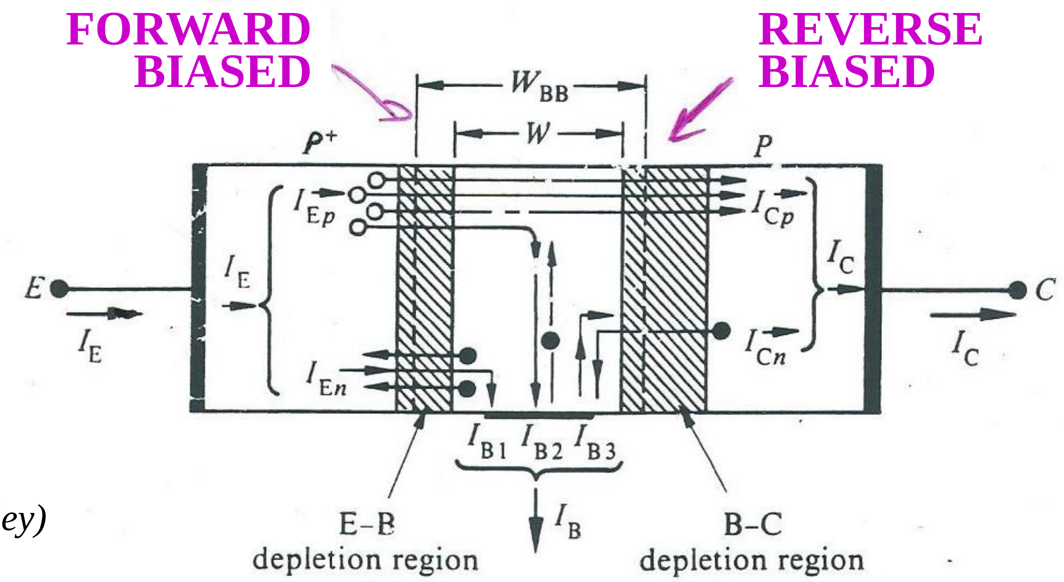
Typically $W < 1\ \mu m$

Figure from "The Bipolar Junction Transistor", G. W. Neudeck (Addison-Wesley)

P^+NP operating in the active region



Figures from "The Bipolar Junction Transistor", G. W. Neudeck (Addison-Wesley)


FORWARD BIASED

REVERSE BIASED

I_{Ep} : injected holes (forward diffusion current) from Emitter to Base

I_{En} : back injected electrons (forward diffusion current) from Base to Emitter ($I_{En} << I_{Ep}$ due to p^+n junction)

R-G currents in D.R.s have been ignored

$$I_E = I_{Ep} + I_{En}$$
$$I_C = I_{Cp} + I_{Cn}$$
$$I_B = I_{B1} + I_{B2} - I_{B3}$$

Practically

$I_{Ep} \approx I_E$ and slightly $> I_C$

$I_B << I_C$ or I_E

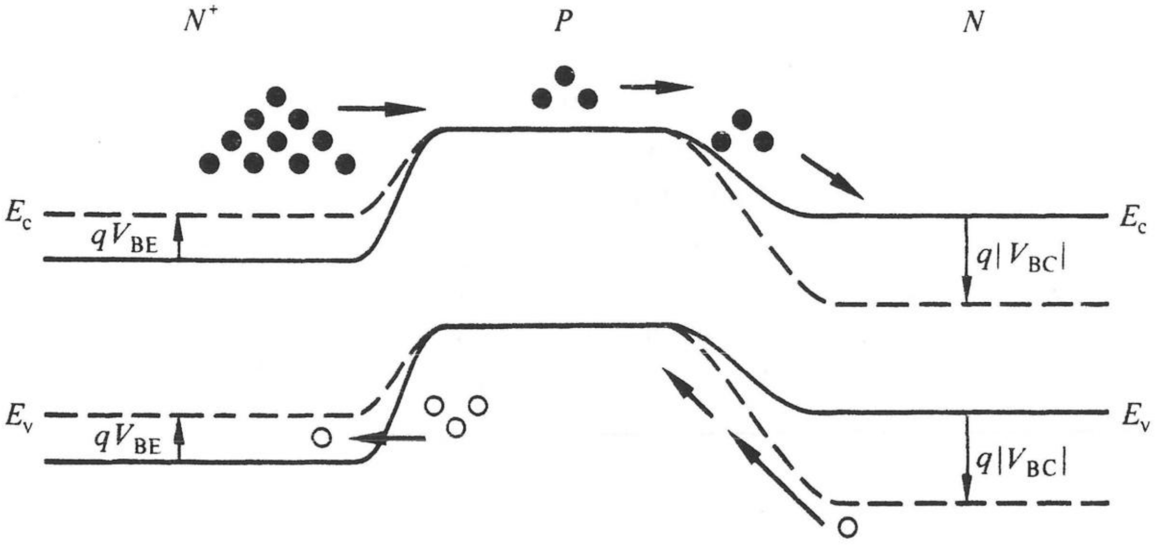
I_{cp} : holes injected from the Emitter that reach the Collector ($I_{Cp} \approx I_{Ep}$ in good transistors)

I_{cn} : reverse current of electrons thermally generated in the Collector near the C-B junction and drifted to the Base

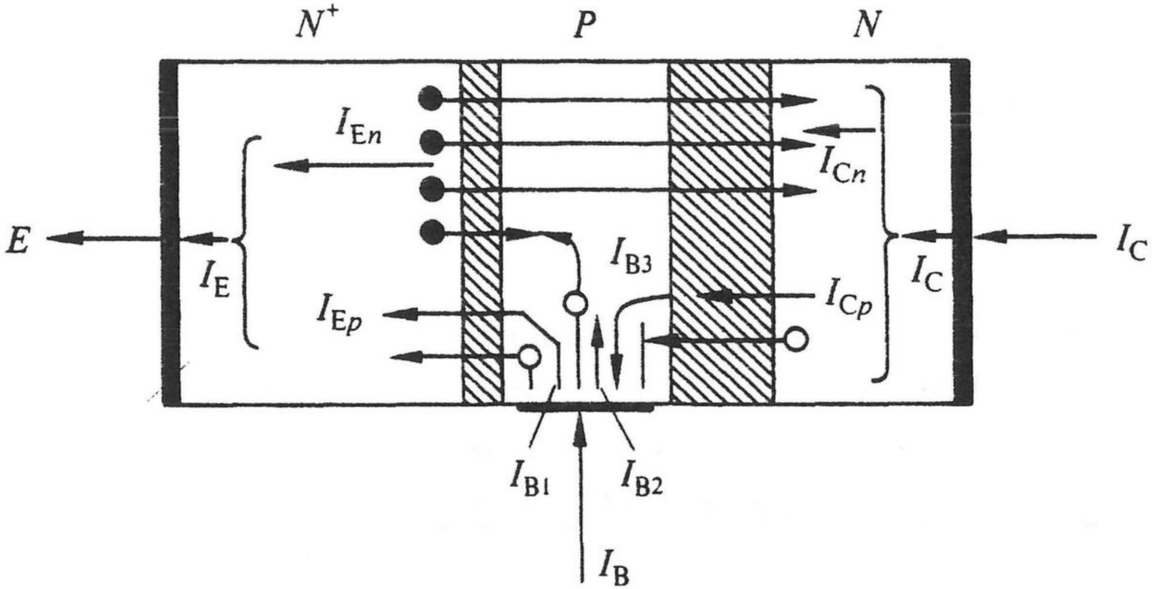
$I_{B1} = I_{En}$ (small due to p^+n) **$I_{B3} = I_{Cn}$ (small reverse current)**

I_{B2} : electrons recombining with some of the injected holes within the Base. Small current due to $W<<L_p$

NPN operating in the active region

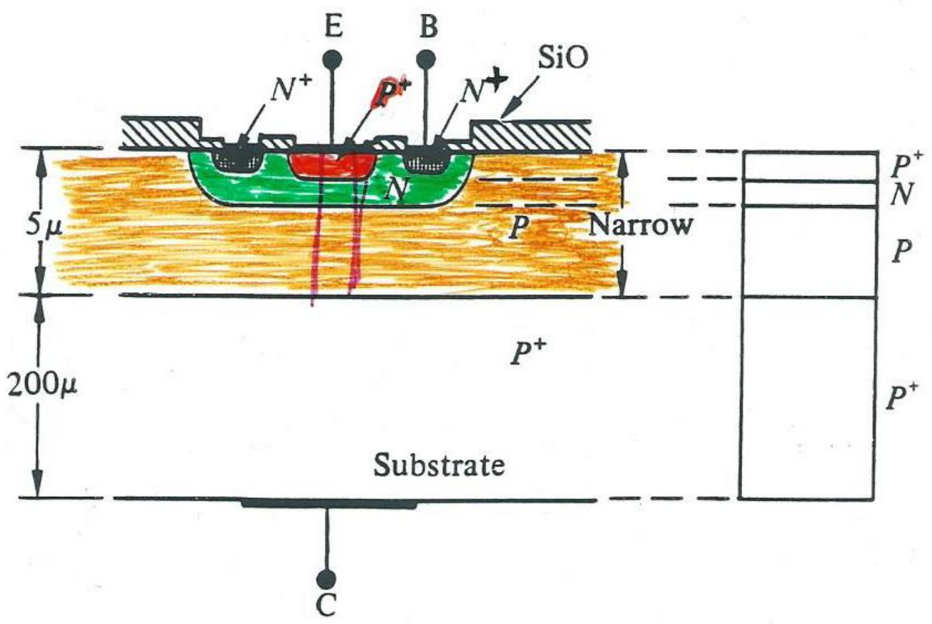


Figures from "The Bipolar Junction Transistor",
G. W. Neudeck (Addison-Wesley)

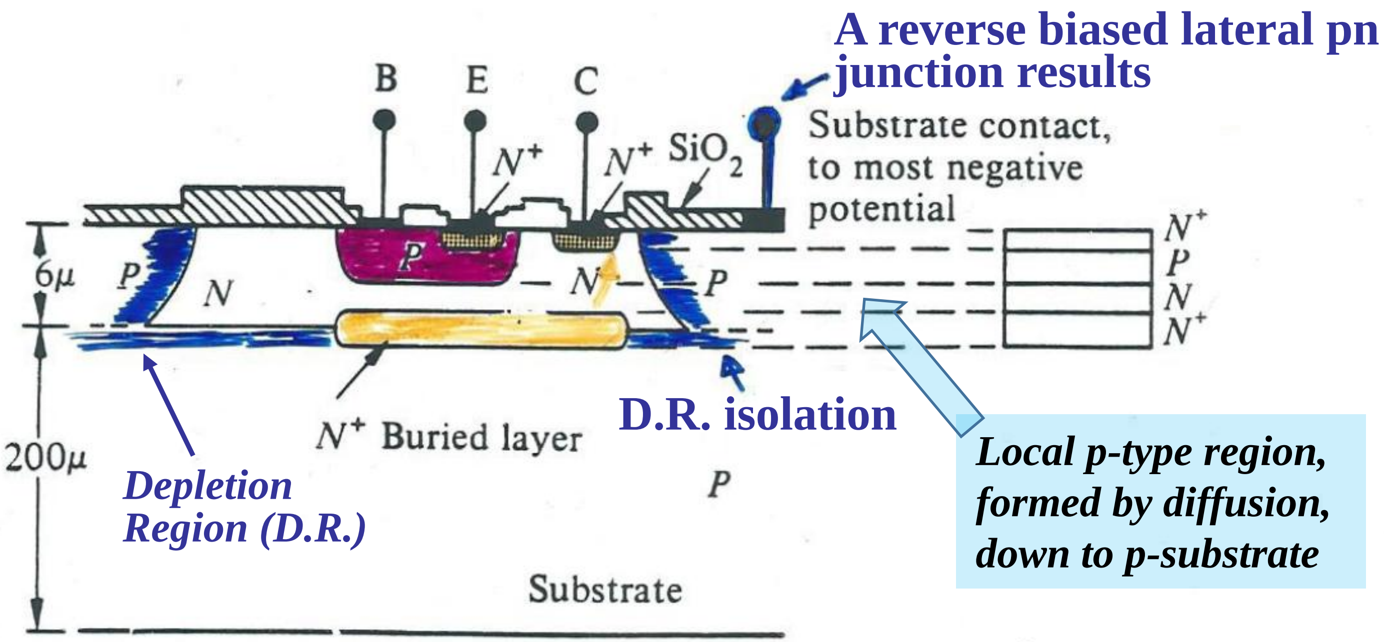


$$I_E = I_{En} + I_{Ep}$$
$$I_C = I_{Cn} + I_{Cp}$$
$$I_B = I_{B1} + I_{B2} - I_{B3}$$

Typical fabrication of Si BJTs



Discrete p$^+$np BJT



A reverse biased lateral pn junction results

D.R. isolation

Depletion Region (D.R.)

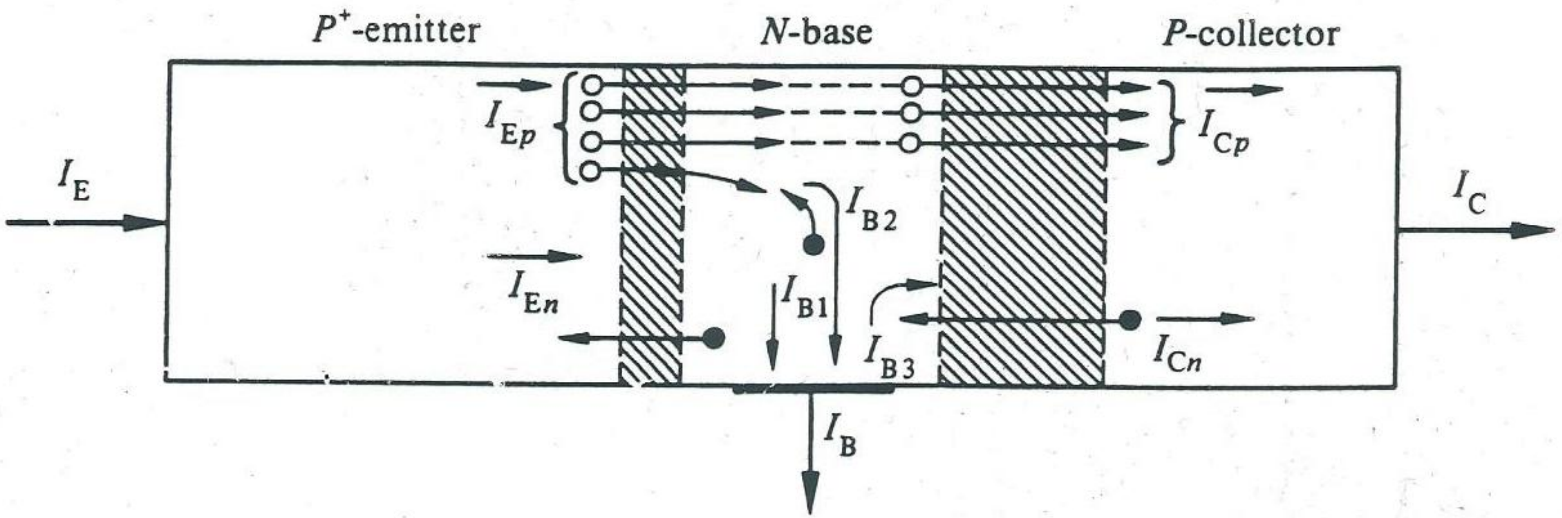
Local p-type region, formed by diffusion, down to p-substrate

Integrated circuit n$^+$pn BJT

Figures from "The Bipolar Junction Transistor", G. W. Neudeck (Addison-Wesley)

Circuit variables and BJT performance parameters

pnp operating in active region



From "The Bipolar Junction Transistor", G. W. Neudeck (Addison-Wesley)

Currents for active region

$$I_E = I_{Ep} + I_{En}$$

$$I_C = I_{Cp} + I_{Cn}$$

Kirchhoff's 1st low $\Rightarrow I_E = I_B + I_C \Rightarrow$

$$I_B = I_E - I_C = I_{B1} + I_{B2} - I_{B3}$$

Base Transport Factor : $\alpha_T = \dfrac{I_{Cp}}{I_{Ep}}$ **In properly designed BJT devices W $\ll L_p \Rightarrow$** $\alpha_T \rightarrow 1.0$

Emitter Injection Efficiency : $\gamma = \dfrac{I_{Ep}}{I_E} = \dfrac{I_{Ep}}{I_{Ep}+I_{En}}$ **If in p$^+$n E-B junction $I_{En} \rightarrow 0 \Rightarrow$** $\gamma \rightarrow 1.0$

Dc alpha : $\alpha_{dc} = \dfrac{I_C}{I_E} = \dfrac{I_{Cp}+I_{Cn}}{I_{Ep}+I_{En}}$ **Considering that $I_{Cp} \gg I_{Cn} \Rightarrow$** $\alpha_{dc} \cong \dfrac{I_{Cp}}{I_{Ep}+I_{En}}$

BJT performance parameters definition

Dc alpha : $\alpha_{dc} \cong \frac{I_{Cp}}{I_{Ep}+I_{En}} = \frac{I_{Cp}}{I_{Ep}}\left[\frac{1}{1+I_{En}/I_{Ep}}\right] = \alpha_T \left[\frac{I_{Ep}}{I_{Ep}+I_{En}}\right] \Rightarrow \alpha_{dc} \cong \gamma\alpha_T$

If $\alpha_T \rightarrow 1.0$ **and** $\gamma \rightarrow 1.0 \Rightarrow \alpha_{dc} \rightarrow 1.0$

Beta : $\beta_{dc} = \frac{I_C}{I_B} = \frac{I_C}{I_E - I_C} \Rightarrow \beta_{dc} = \frac{I_C}{I_E(1-I_C/I_E)} = \frac{I_C/I_E}{1-I_C/I_E} \Rightarrow \beta_{dc} = \frac{\alpha_{dc}}{1-\alpha_{dc}}$

If $\alpha_{dc} \rightarrow 1.0 \Rightarrow \beta_{dc} \rightarrow \infty$

Beta **is an important parameter, representing the current gain capability of the transistor**

Output versus input current equations

For PNP operating in active region

Common Base active region

$$I_C = I_{Cp} + I_{Cn} = \alpha_T I_{Ep} + I_{Cn} = \gamma \alpha_T \frac{I_{Ep}}{\gamma} + I_{Cn} = \alpha_{dc} I_E + I_{Cn} \Rightarrow I_C = \alpha_{dc} I_E + I_{BCO} \quad (1)$$

where $I_{BCO} = I_{Cn}$ is the reverse current flowing through the C-B junction when $I_E = 0$ (open circuit for the emitter / input)

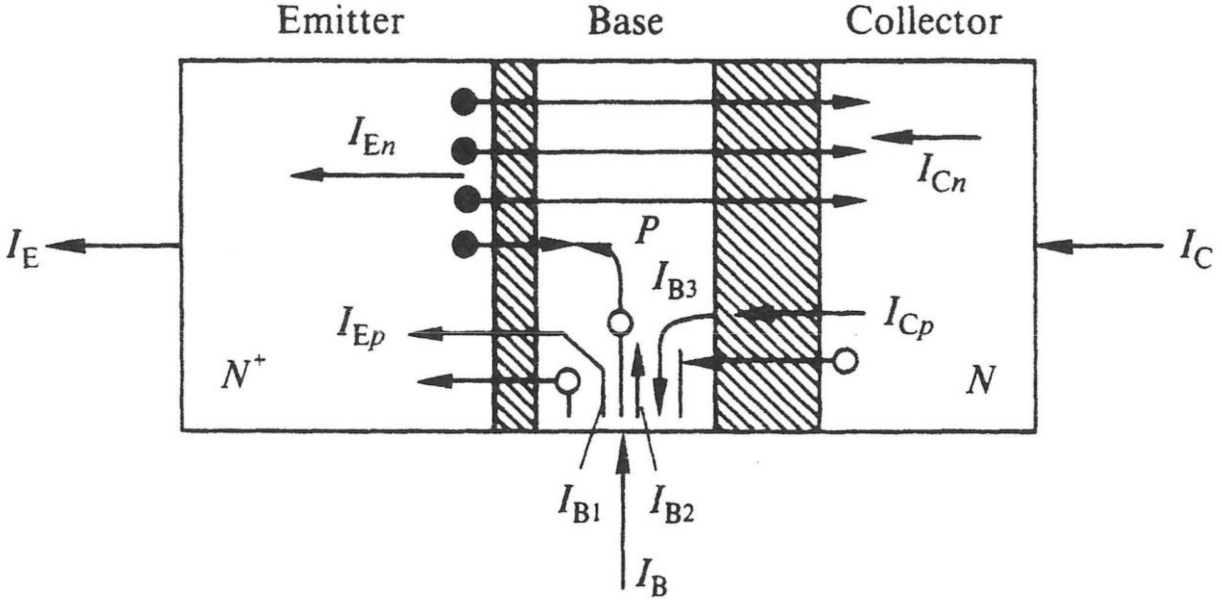
Common Emitter active region

Eq. (1) $\Rightarrow I_C = \alpha_{dc}(I_B + I_C) + I_{BCO} \Rightarrow I_C(1 - \alpha_{dc}) = \alpha_{dc} I_B + I_{BCO} \Rightarrow I_C = \frac{\alpha_{dc}}{(1-\alpha_{dc})} I_B + \frac{I_{BCO}}{(1-\alpha_{dc})} =$
$\beta_{dc} I_B + I_{BCO}(\beta_{dc} + 1) \Rightarrow I_C = \beta_{dc} I_B + I_{ECO}$ (2)

where $I_{ECO} = I_{BCO}(\beta_{dc} + 1)$ is the current flowing through the three regions of the transistor when $I_B = 0$ (open circuit for the base / input)

Circuit variables and performance parameters for NPN

npn operating in active region



From "The Bipolar Junction Transistor", G. W. Neudeck (Addison-Wesley)

Currents for active region

$$I_E = I_{En} + I_{Ep}$$

$$I_C = I_{Cn} + I_{Cp}$$

$$I_B = I_E - I_C = I_{B1} + I_{B2} - I_{B3}$$

where $I_{B1} = I_{Ep}$ **and** $I_{B3} = I_{Cp}$

***Base Transport Factor*:** $\alpha_T = \frac{I_{Cn}}{I_{En}}$

***Emitter Injection Efficiency*:** $\gamma = \frac{I_{En}}{I_E} = \frac{I_{En}}{I_{En}+I_{Ep}}$

***Dc alpha*:** $\alpha_{dc} = \frac{I_C}{I_E} \cong \frac{I_{Cn}}{I_{En}+I_{Ep}}$

***Beta*:** $\beta_{dc} = \frac{I_C}{I_B}$

Common Base active region

$I_C = \alpha_{dc} I_E + I_{CBO}$ **where** $I_{CBO} = I_{Cp}$

Common Emitter active region

$I_C = \beta_{dc} I_B + I_{CEO}$

with $I_{CEO} = I_{CBO}(\beta_{dc} + 1)$

Analysis of ideal BJT

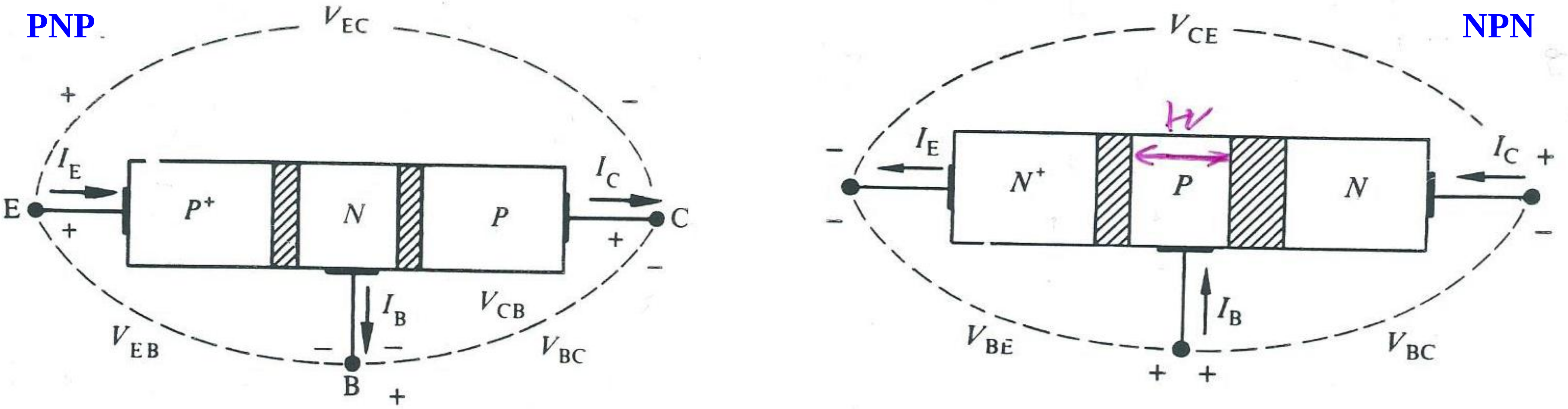
Objective: Derive relationships between the terminal currents and the voltages applied at E-B and C-B junctions, for steady-state condition

Assumptions:

- **One-dimensional structure and**
- **Low level injection, zero electric field in the quasi-neutral bulk regions and base bulk region width W<< $L_{p(n)}$**
- **No recombination-generation in the quasi-neutral Base and the two Depletion Regions for an Ideal BJT**

Current and voltage definitions

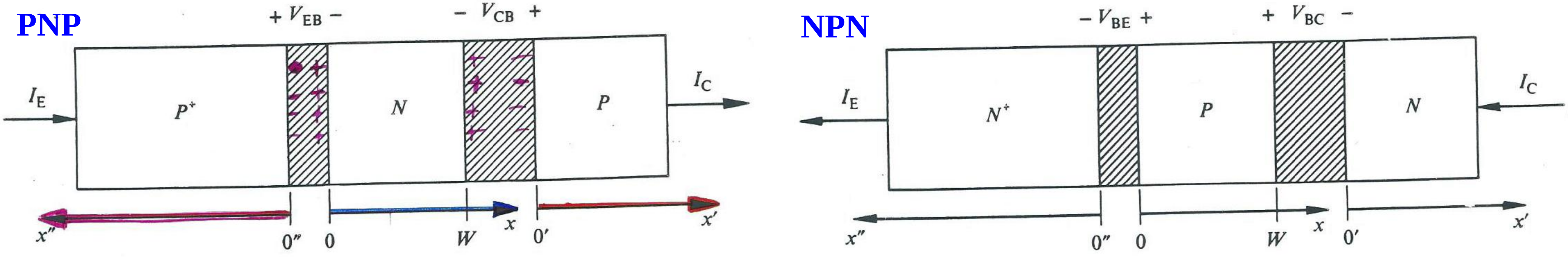
PNP **NPN**



Figures from "The Bipolar Junction Transistor", G. W. Neudeck (Addison-Wesley)

Coordinate axes and material parameters

UoC Physics



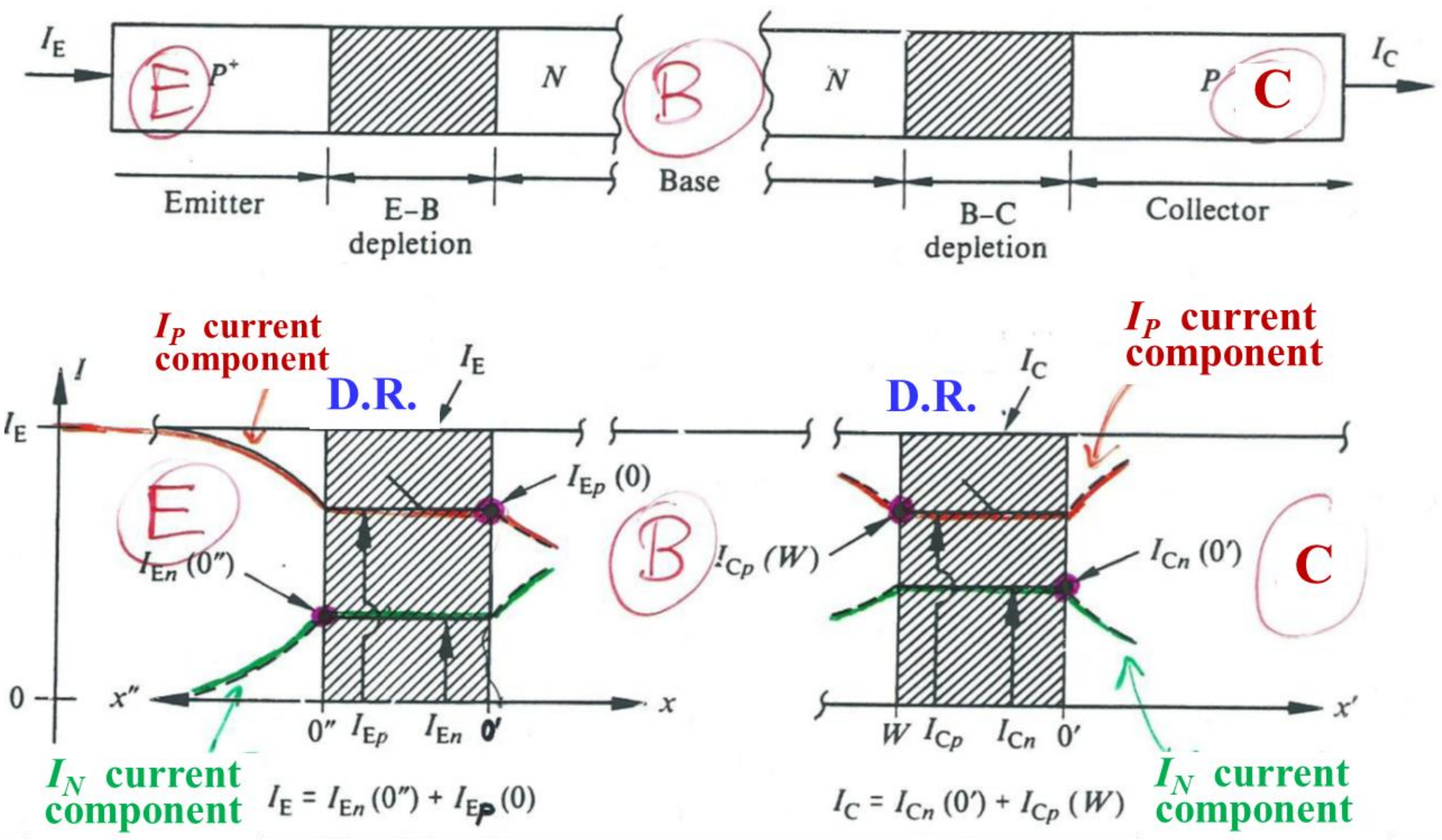
$N_{AE} = N_E$	$N_{DB} = N_B$	$N_{AC} = N_C$	$N_{DE} = N_E$	$N_{AB} = N_B$	$N_{DC} = N_C$
$L_N = L_E$	$L_P = L_B$	$L_N = L_C$	$L_P = L_E$	$L_N = L_B$	$L_P = L_C$
$D_N = D_E$	$D_P = D_B$	$D_N = D_C$	$D_P = L_E$	$D_N = D_B$	$D_P = D_C$
$n_{p0} = n_{E0}$	$p_{n0} = p_{B0}$	$n_{p0} = n_{C0}$	$p_{n0} = p_{E0}$	$n_{p0} = n_{B0}$	$p_{n0} = p_{C0}$
$\tau_n = \tau_E$	$\tau_p = \tau_B$	$\tau_n = \tau_C$	$\tau_p = \tau_E$	$\tau_n = \tau_B$	$\tau_p = \tau_C$

Figures from "The Bipolar Junction Transistor", G. W. Neudeck (Addison-Wesley)

Steps for deriving the steady-state I-V relationships:

- **Determination of the minority carrier concentration distributions in the bulk regions of E, B, C → solution of minority carrier diffusion equations**
- **Determination of the terminal currents I_E, I_B, I_C from current components determined as minority carrier diffusion currents at the depletion region edges**

Current components in a PNP BJT



From "The Bipolar Junction Transistor", G. W. Neudeck (Addison-Wesley)

$$I_E = I_{Ep}(0) + I_{En}(0'')$$

$$I_{Ep}(0) = -qAD_B \frac{d\,\Delta p_B}{dx}\bigg|_{x=0}$$

$$I_{En}(0'') = -qAD_E \frac{d\,\Delta n_E}{dx''}\bigg|_{x''=0}$$

The formula for I_{En} corresponds to the correct positive sign for the x-axis

$$I_C = I_{Cp}(W) + I_{Cn}(0')$$

$$I_{Cp}(W) = -qAD_B \frac{d\,\Delta p_B}{dx}\bigg|_{x=W}$$

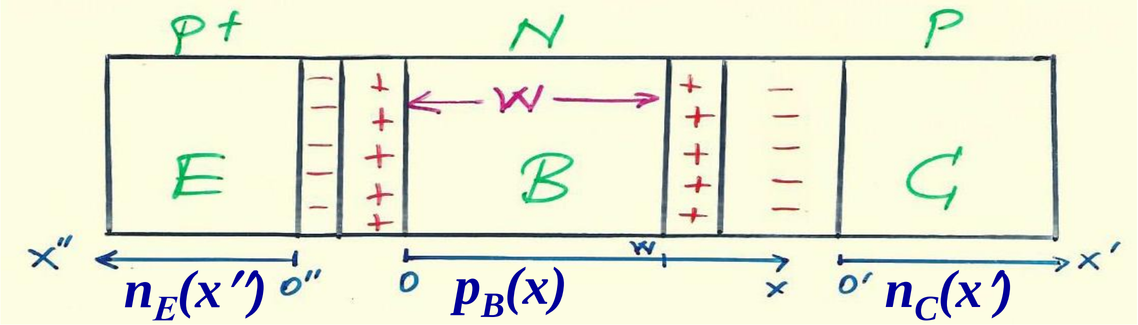
$$I_{Cn}(0') = qAD_C \frac{d\,\Delta n_C}{dx'}\bigg|_{x'=0}$$

$$I_B = I_E - I_C$$

Minority carrier diffusion equations in a PNP BJT

UoC Physics

$n_E(x'')$, $p_B(x)$ and $n_E(x')$ should be determined in the bulk E, B, and C regions



Base → $$\boxed{\frac{d^2\Delta p_B(x)}{dx^2} = \frac{\Delta p_B(x)}{D_B\tau_B} = \frac{\Delta p_B(x)}{L_B^2}}$$ where $\tau_p = \tau_B$, $L_B = \sqrt{D_B\cdot\tau_B}$

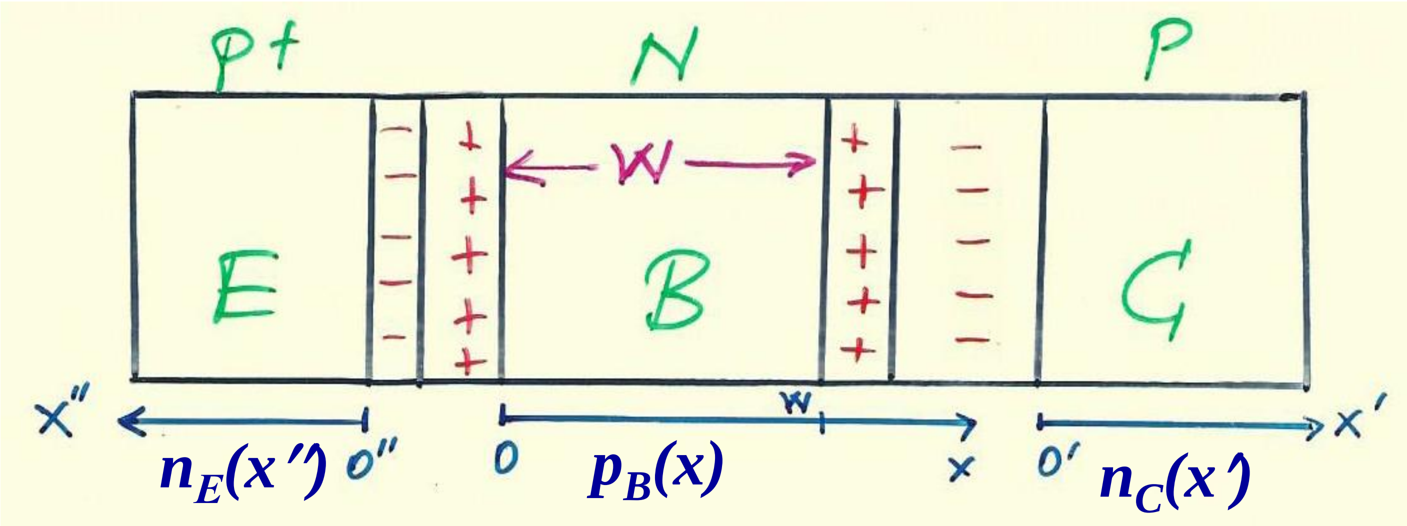
→ **General solution** $$\Delta p_B(x) = C_1 e^{x/L_B} + C_2 e^{-x/L_B}$$

C_1 and C_2 from two boundary conditions

Emitter → $$\frac{d^2\Delta n_E(x'')}{dx''^2} = \frac{\Delta n_E(x'')}{L_E^2} \Rightarrow \Delta n_E(x'') = C_1 e^{x''/L_E} + C_2 e^{-x''/L_E}$$

Collector → $$\frac{d^2\Delta n_C(x')}{dx'^2} = \frac{\Delta n_C(x')}{L_C^2} \Rightarrow \Delta n_C(x') = C_1 e^{x'/L_C} + C_2 e^{-x'/L_C}$$

Boundary conditions for minority carriers in PNP



$$\Delta n_E(x''=\infty) = 0$$

$$\Delta n_C(x'=\infty) = 0$$

$$\Delta n_E(0'') = n_{E0}\left(e^{qV_{EB}/kT}-1\right)$$

$$\Delta p_B(0) = p_{B0}\left(e^{qV_{EB}/kT}-1\right)$$

$$\Delta p_B(W) = p_{B0}\left(e^{qV_{CB}/kT}-1\right)$$

$$\Delta n_C(0') = n_{C0}\left(e^{qV_{CB}/kT}-1\right)$$

where

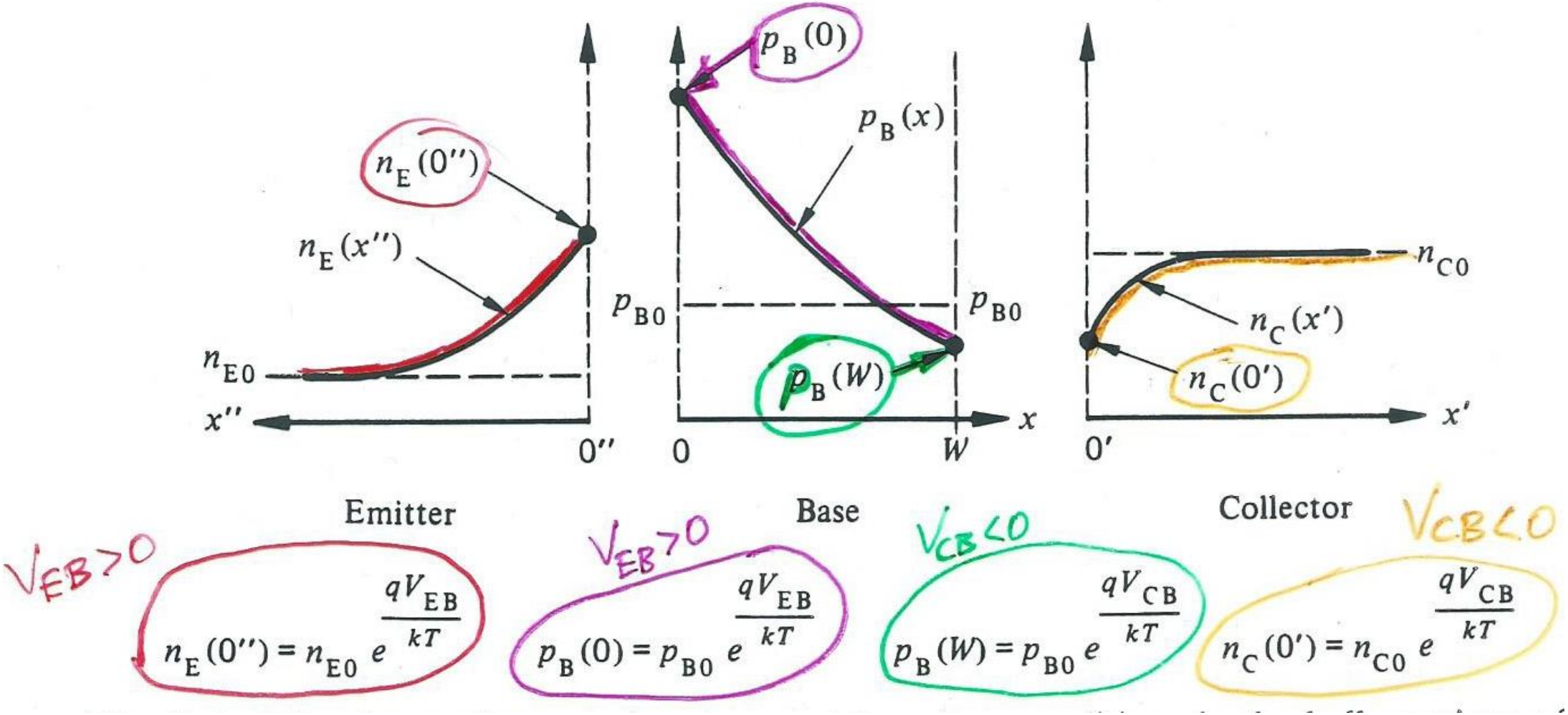
$n_{E0} = n_{p0}$ in the p-type Emitter

$p_{B0} = p_{n0}$ in the n-type Base

$n_{E0} = n_{p0}$ in p-type Collector

PNP BJT operation at forward active region

Distribution of minority carriers in a PNP BJT operating in the active region



$V_{EB} > 0$

$$n_E(0'') = n_{E0}\, e^{\frac{qV_{EB}}{kT}}$$

$V_{EB} > 0$

$$p_B(0) = p_{B0}\, e^{\frac{qV_{EB}}{kT}}$$

$V_{CB} < 0$

$$p_B(W) = p_{B0}\, e^{\frac{qV_{CB}}{kT}}$$

$V_{CB} < 0$

$$n_C(0') = n_{C0}\, e^{\frac{qV_{CB}}{kT}}$$

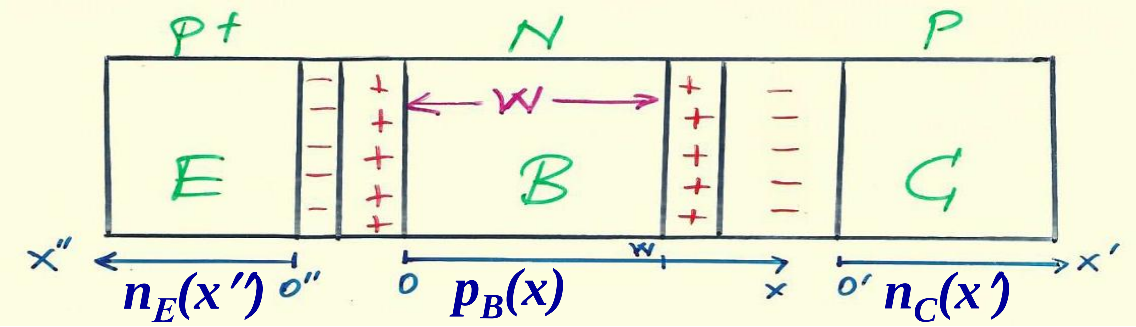
From "The Bipolar Junction Transistor", G. W. Neudeck (Addison-Wesley)

Distribution of holes in the Base w/o recombination

W << L_B in the Base (e.g. W < 1μm, $L_B \sim$ *20-30 μm*)

⇒ NO RECOMBINATION IN THE BASE

$$\Rightarrow \quad \frac{d^2\Delta p_B(x)}{dx^2} \cong 0 \quad \Rightarrow$$



$$\Delta p_B(x) = C_1 \cdot x + C_2$$

$$\left.\begin{array}{l} \Delta p_B(0) = C_1 \cdot 0 + C_2 = C_2 \\ \Delta p_B(W) = C_1 \cdot W + C_2 = C_1 \cdot W + \Delta p_B(0) \end{array}\right\} \Rightarrow C_1 = \frac{\Delta p_B(W) - \Delta p_B(0)}{W}$$

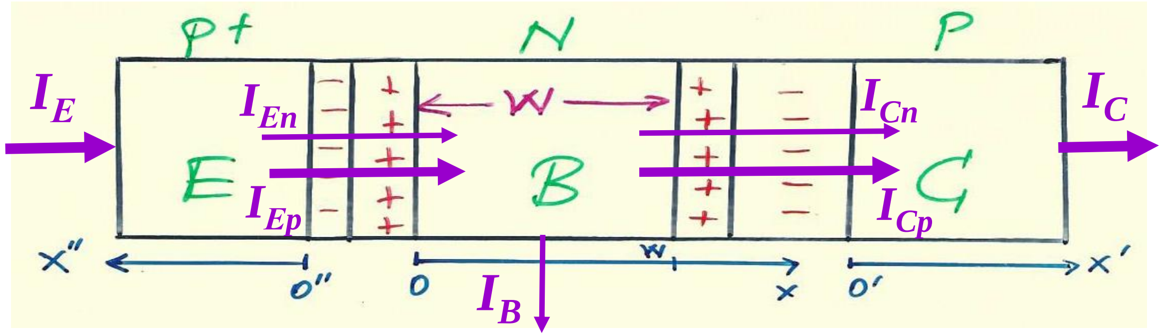
$$\Rightarrow \quad \Delta p_B(x) = -\left[\frac{\Delta p_B(0) - \Delta p_B(W)}{W}\right]x + \Delta p_B(0)$$

$$\boxed{\Delta p_B(x) = -\left[\frac{p_{B0}\left(e^{qV_{EB}/kT}-1\right) - p_{B0}\left(e^{qV_{CB}/kT}-1\right)}{W}\right]x + p_{B0}\left(e^{qV_{EB}/kT}-1\right)} \quad ①$$

$$\boxed{I_{E_p}(0) = -qAD_B \left.\frac{\Delta p_B}{dx}\right|_{x=0} = \frac{qAD_B}{W}\, p_{B0}\left[\left(e^{qV_{EB}/kT}-1\right) - \left(e^{qV_{CB}/kT}-1\right)\right]} \quad ②$$

Distribution of electrons in the p-type Emitter & I_E

$$I_{E_p}(0) = -qAD_B \frac{\Delta P_B}{dx}\Big|_{x=0} = \frac{qAD_B}{W} P_{B0}\left[\left(e^{qV_{EB}/kT}-1\right)-\left(e^{qV_{CB}/kT}-1\right)\right] \quad ②$$



$$I_E = I_{E_p}(0) + I_{E_n}(0'')$$

$$I_E = -qAD_B \frac{d\Delta P_B}{dx}\Big|_{x=0} - qAD_E \frac{d\Delta n_E}{dx''}\Big|_{x''=0} \quad (3)$$

change of sign to minus to give the correct I_{En} direction, since the x′′-axis is opposite to the x-axis

$$\frac{d^2\Delta n_E(x'')}{dx''^2} = \frac{\Delta n_E(x'')}{L_E^2} \Rightarrow \Delta n_E(x'') = C_1 e^{-x''/L_E} + C_2 e^{x''/L_E}$$

Boundary conditions

$$\Delta n_E(x''=\infty) = 0 \Rightarrow C_2 = 0$$

$$\Delta n_E(0'') = C_1 e^{0''} = C_1 = n_{E0}\left(e^{qV_{EB}/kT}-1\right)$$

$$\Rightarrow \boxed{\Delta n_E(x'') = n_{E0}\left(e^{qV_{EB}/kT}-1\right) e^{-x''/L_E}} \quad ④$$

$$I_{E_n}(0'') = -qAD_E \frac{d\Delta n_E}{dx''}\Big|_{x''=0} \overset{④}{=} \frac{qAD_E}{L_E} n_{E0}\left(e^{qV_{EB}/kT}-1\right) \quad ⑤$$

$$⑥$$

$$②+⑤ \overset{③}{\Longrightarrow} \boxed{I_E = qA\left[\frac{D_E n_{E0}}{L_E} + \frac{D_B P_{B0}}{W}\right]\left(e^{\frac{qV_{EB}}{kT}}-1\right) - \left[\frac{qAD_B}{W} P_{B0}\right]\left(e^{\frac{qV_{CB}}{kT}}-1\right)}$$

Solutions for I_C and I_B currents

$$\boxed{I_C = I_{Cp}(W) + I_{Cn}(0')} \overset{\textcircled{7}}{=} -qAD_B \frac{d\Delta p_B}{dx}\Big|_{x=W} + qAD_C \frac{d\Delta n_C}{dx'}\Big|_{x'=0}$$

We can proceed in a similar way as in the previous calculations. However, the approximation of IDEAL PNP BJT $\Rightarrow$ $I_{Ep}(0) = I_{Ep}(W) = I_{Cp}(W)$ **(8)**

$I_{Cn}(0')$**, given by the second part of Eq. (7), can be derived from Eq. (5) without the sign reversal and by replacing** C **with** E **and** V_{CB} **with** V_{EB} **(x′-axis has same direction with x-axis)**

$$I_{En}(0'') = -qAD_E \frac{d\Delta n_E}{dx''}\Big|_{x''=0} = \frac{qAD_E}{L_E} n_{E0}\left(e^{qV_{EB}/kT} - 1\right) \quad \textcircled{5}$$

$$\rightarrow \boxed{I_{Cn} = \frac{-qAD_C}{L_C} n_{C0}\left(e^{qV_{CB}/kT} - 1\right)} \quad \textcircled{9}$$

$$\textcircled{7} \xrightarrow{\textcircled{9}\,\textcircled{8}} \boxed{I_C = \left[\frac{qAD_B}{W} p_{B0}\right]\left(e^{qV_{EB}/kT} - 1\right) - qA\left[\frac{D_C n_{C0}}{L_C} + \frac{D_B p_{B0}}{W}\right]\left(e^{qV_{CB}/kT} - 1\right)} \quad \textcircled{10}$$

$$I_B = I_E - I_C \Rightarrow \boxed{I_B = \frac{qAD_E}{L_E} n_{E0}\left(e^{qV_{EB}/kT} - 1\right) + \frac{qAD_C}{L_C} n_{C0}\left(e^{qV_{CB}/kT} - 1\right)} \quad \textcircled{11}$$

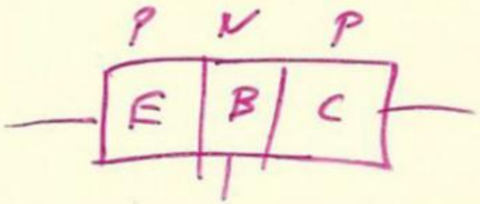
The current relationships of the Ideal pnp BJT

Since

$$n_{E0} = n_i^2/N_{AE} = n_i^2/N_E$$

$$p_{B0} = n_i^2/N_{DB} = n_i^2/N_B$$

$$n_{C0} = n_i^2/N_{AC} = n_i^2/N_C$$

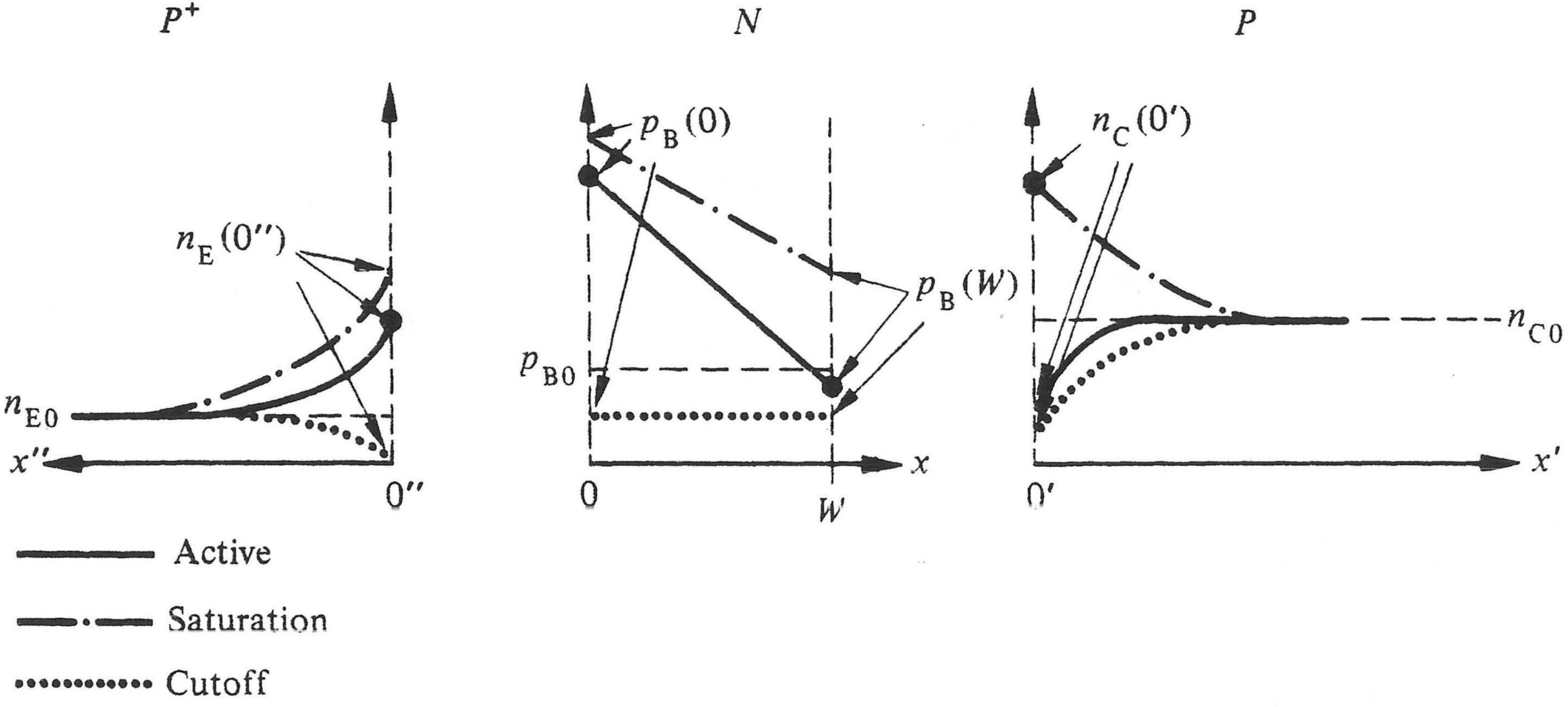


$$I_E = qAn_i^2\left[\frac{D_E}{L_E N_E} + \frac{D_B}{WN_B}\right]\left(e^{qV_{EB}/KT} - 1\right) - qAn_i^2\left[\frac{D_B}{WN_B}\right]\left(e^{\frac{qV_{CB}}{KT}} - 1\right)$$ ⑫

$$I_C = qAn_i^2\left[\frac{D_B}{WN_B}\right]\left(e^{\frac{qV_{EB}}{KT}} - 1\right) - qAn_i^2\left[\frac{D_C}{L_C N_C} + \frac{D_B}{WN_B}\right]\left(e^{qV_{CB}/KT} - 1\right)$$ ⑬

$$I_B = qAn_i^2\left[\frac{D_E}{L_E N_E}\right]\left(e^{\frac{qV_{EB}}{KT}} - 1\right) + qAn_i^2\left[\frac{D_C}{L_C N_C}\right]\left(e^{qV_{CB}/KT} - 1\right)$$ ⑭

Distribution of minority carriers in Ideal pnp BJT



From "The Bipolar Junction Transistor", G. W. Neudeck (Addison-Wesley)

Active region operation – pnp currents I

$$V_{EB} > 0 \rightarrow \exp\left(\frac{qV_{EB}}{KT}\right) >> 1$$

$$V_{CB} < 0 \rightarrow \exp\left(\frac{qV_{CB}}{KT}\right) << 1$$

For voltages > few KT/q≈0.026 V at 300K

$$(12) \rightarrow \boxed{I_E \simeq qAn_i^2\left[\frac{D_E}{L_E N_E} + \frac{D_B}{W N_B}\right]\left(e^{qV_{EB}/KT}\right)} \quad (15) \leftarrow$$

It is assumed that $qAn_i^2(D_E/L_EN_E) \approx 0$

$$(2) \rightarrow I_{Ep} \simeq qAn_i^2 \frac{D_B}{W N_B} e^{qV_{EB}/KT} = I_{Cp} \quad (\text{ΙΔΑΝΙΚΟ BJT})$$

$$(5) \rightarrow I_{En} \simeq qAn_i^2 \frac{D_E}{L_E N_E}\left(e^{qV_{EB}/KT} \cancel{-1}\right) = I_{B1}$$

$$(13) \rightarrow I_C \simeq qAn_i^2\left[\frac{D_B}{W N_B}\right]e^{qV_{EB}/KT} - \underbrace{qAn_i^2\left[\frac{D_C}{L_C N_C}\right]}_{= I_{Cn} = I_{B3}}$$

$$\Rightarrow \boxed{I_C \simeq qAn_i^2 \frac{D_B}{W N_B} e^{qV_{EB}/KT}} \quad (16)$$

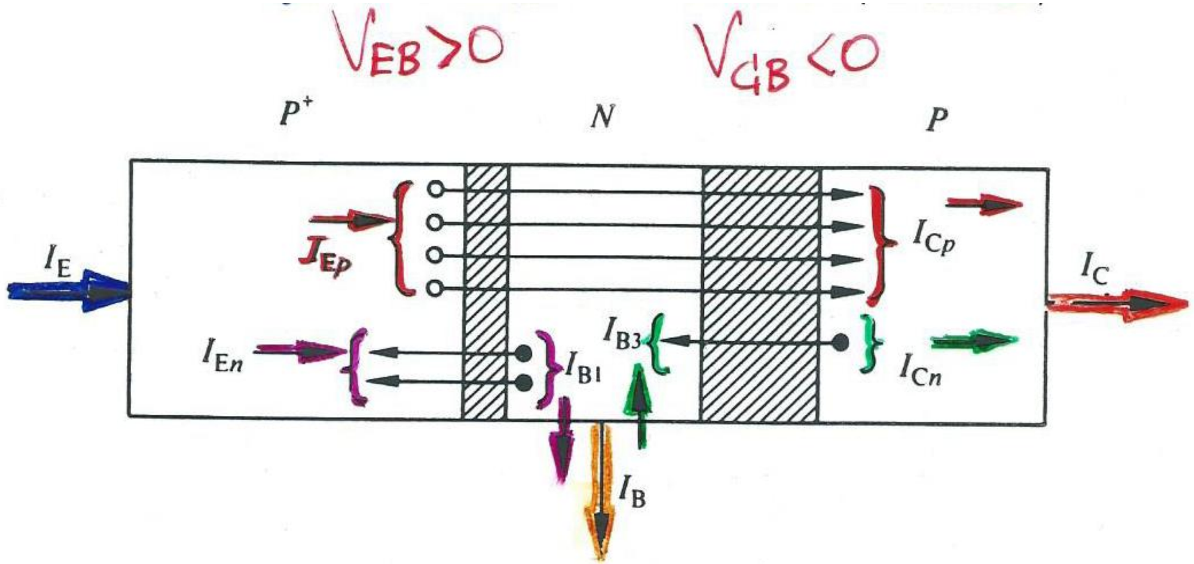
$$(14) \rightarrow I_B \simeq \underbrace{qAn_i^2 \frac{D_E}{L_E N_E} e^{qV_{EB}/KT}}_{I_{En}} - \underbrace{\frac{qAn_i^2 D_C}{L_C N_C}}_{I_{Cn} =} \rightarrow \boxed{I_B \simeq qAn_i^2 \frac{D_E}{L_E N_E} e^{qV_{EB}/KT}} \quad (17)$$

I_{B3}

Active region operation – pnp currents II

Forward Bias

Reverse Bias



$$I_E \simeq I_{En} + I_{Ep} \simeq qAn_i^2\left[\frac{D_E}{L_E N_E} + \frac{D_B}{W N_B}\right]e^{qV_{EB}/kT}$$

$$I_C \simeq qAn_i^2\,\frac{D_B}{W N_B}\,e^{qV_{EB}/kT}$$

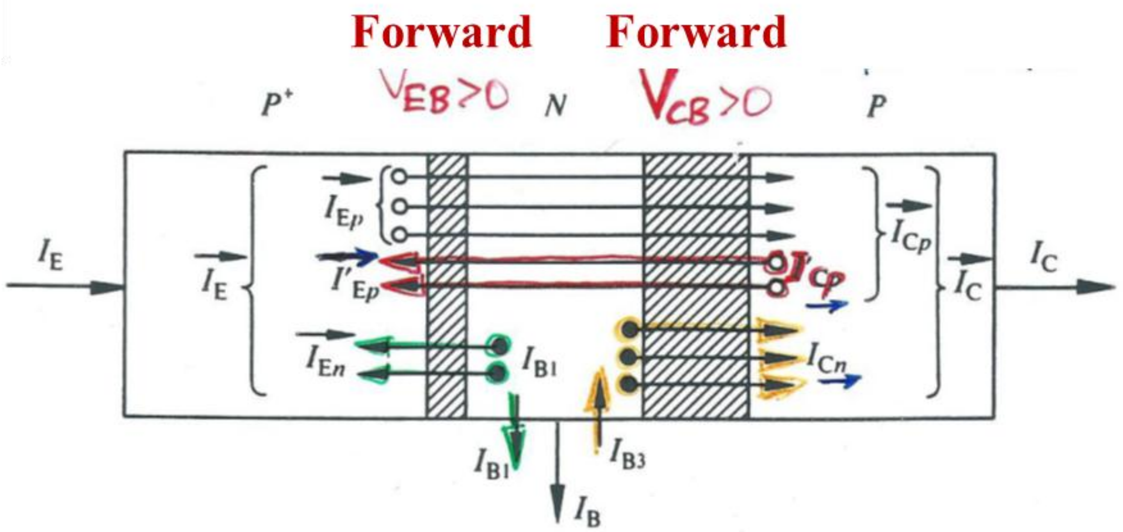
$$I_B \simeq qAn_i^2\,\frac{D_E}{L_E N_E}\,e^{qV_{EB}/kT}$$

$$I_C \approx I_{Cp}$$

$$I_B \approx I_{B1} = I_{En}$$

From "The Bipolar Junction Transistor", G. W. Neudeck (Addison-Wesley)

Saturation region operation – pnp currents



From "The Bipolar Junction Transistor", G. W. Neudeck (Addison-Wesley)

$$\left.\begin{array}{l} V_{EB}>0 \\ V_{CB}>0 \end{array}\right\} \Rightarrow e^{qV_{EB}/kT} \gg 1 \quad \textbf{and} \quad e^{qV_{CB}/kT} \gg 1$$

$$I_E = qAn_i^2\left[\frac{D_E}{L_EN_E}+\frac{D_B}{WN_B}\right]\left(e^{qV_{EB}/kT}\cancel{-1}\right) - qAn_i^2\left[\frac{D_B}{WN_B}\right]\left(e^{qV_{CB}/kT}\cancel{-1}\right)$$

$$I_E \simeq \underbrace{qAn_i^2\frac{D_E}{L_EN_E}e^{qV_{EB}/kT}}_{I_{En}} + \underbrace{qAn_i^2\frac{D_B}{WN_B}e^{qV_{EB}/kT}}_{I_{Ep}} \underbrace{- qAn_i^2\frac{D_B}{WN_B}e^{qV_{CB}/kT}}_{I'_{Ep}=I'_{Cp}}$$

$$I_C \simeq \underbrace{qAn_i^2\left[\frac{D_B}{WN_B}\right]e^{qV_{EB}/kT}}_{I_{Ep}} + \underbrace{- qAn_i^2\frac{D_C}{L_CN_C}e^{qV_{CB}/kT}}_{I_{Cn}\ \text{(negative)}} + \underbrace{- qAn_i^2\frac{D_B}{WN_B}e^{qV_{CB}/kT}}_{I'_{Cp}\ \text{(negative)}}$$

$$I_B \simeq \underbrace{qAn_i^2\frac{D_E}{L_EN_E}e^{qV_{EB}/kT}}_{I_{B_1}=I_{En}} + \underbrace{qAn_i^2\frac{D_C}{L_CN_C}e^{qV_{CB}/kT}}_{-I_{B_3}=-I_{Cn}}$$

I_E and I_C may become negative for $V_{CB} > V_{EB}$

$I_C < 0 \Rightarrow$ operation in inverted saturation region

Saturation region operation – pnp currents

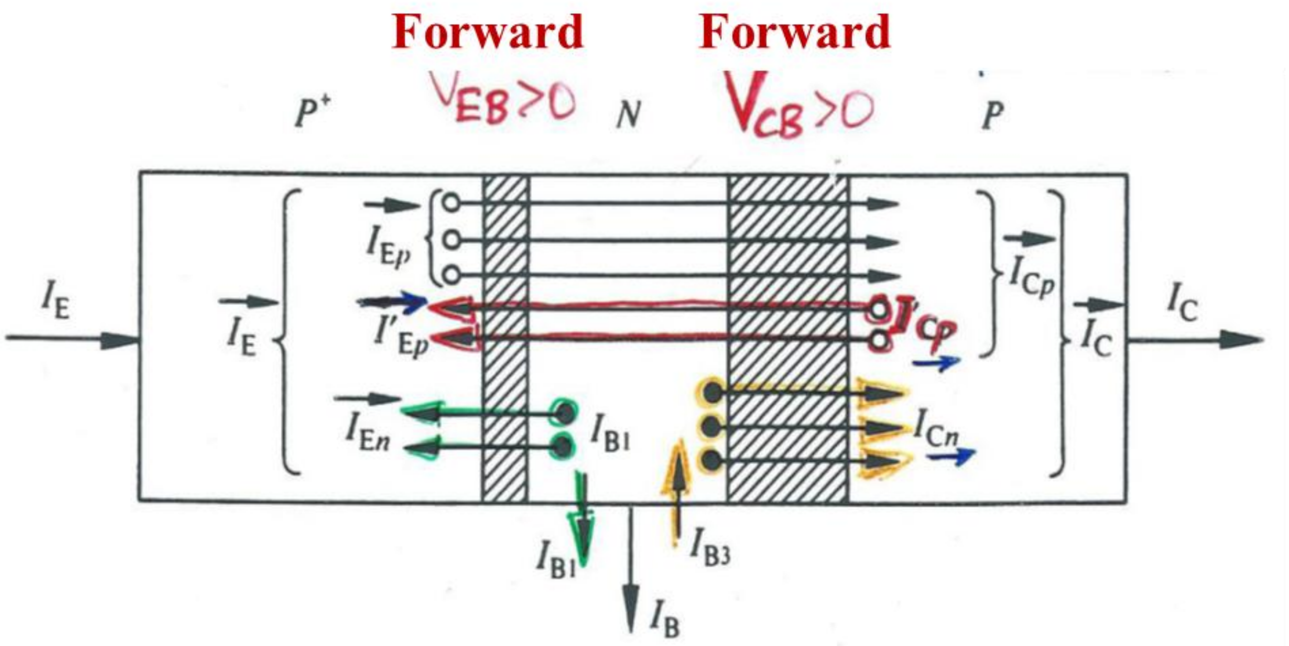


Figure from "The Bipolar Junction Transistor", G. W. Neudeck (Addison-Wesley)

<u>In Saturation region</u>: The junction C-B is forward biased and holes of the p-type Collector are injected to the Base and then pass to the Emitter. This corresponds to current $I'_{Cp} = I'_{Ep}$ of reverse direction compared to $I_{Ep} = I_{Cp}$

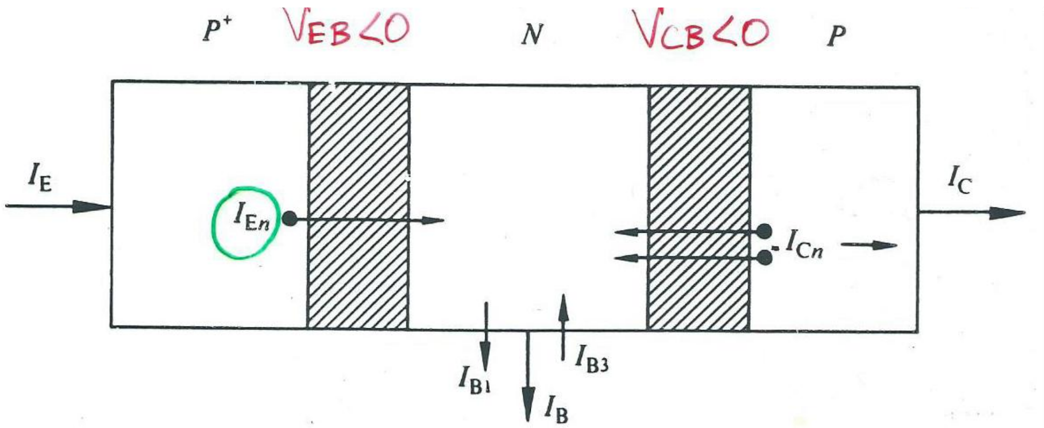
I_B is much larger compared to operation in the active region. I_{B1} is similar but $-I_{B3}$ (> 0) becomes a large additional current component. The Base contact supplies electrons that are injected (forward bias) in the two junctions

I_E and I_C may become negative for $V_{CB} > V_{EB}$

$I_C < 0 \Rightarrow$ operation in inverted saturation region

Cuttoff region operation – pnp currents

Reverse **Reverse**



From "The Bipolar Junction Transistor", G. W. Neudeck (Addison-Wesley)

$$\left.\begin{array}{l} V_{EB}<0 \\ V_{CB}<0 \end{array}\right\} \Rightarrow e^{qV_{EB}/kT} \ll 1 \quad \text{και} \quad e^{qV_{CB}/kT} \ll 1$$

Why are there only the reverse current components I_{En} and I_{Cn} :

Due to $W \ll L_B$ the Base is fully depleted of minority carriers (holes for pnp)

$$\Delta p_n(x) = p_{no}\left(e^{\frac{qV_A}{kT}} - 1\right)e^{-x/L_P}$$

Since $V_A \ll 0 \Rightarrow \Delta p_n(x) \approx -p_{no}e^{-x/L_P}$

E.g. at x=L$_p$/10 → $\Delta p_n(\frac{L_P}{10}) = -p_{no}e^{-1/10} \cong -0.90p_{no}$

$I_E < 0$

additional terms if $W >> L_B$

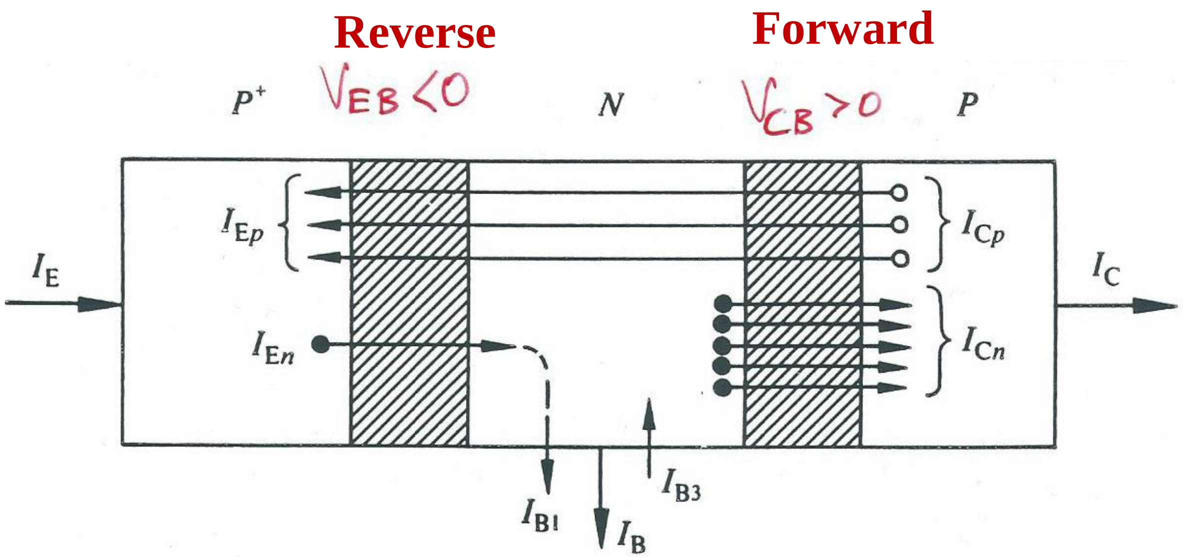
or $\left(-qAn_i^2\frac{D_B}{L_BN_B}\right)$

$$I_E \simeq -qAn_i^2\frac{D_E}{L_EN_E}\left(-P_B\cdot\frac{D_B}{L_B}qA\right)$$

$$I_C \simeq qAn_i^2\frac{D_C}{L_CN_C}\left(+qAn_i^2\frac{D_B}{L_BN_B}\right)$$

$$I_B \simeq -qAn_i^2\frac{D_E}{L_EN_E} - qAn_i^2\frac{D_C}{L_CN_C}$$

Inverted active region operation – pnp currents



From "The Bipolar Junction Transistor", G. W. Neudeck (Addison-Wesley)

For operation in the inverted active region, performance parameters γ_R, α_{dcR} and β_{dcR} are defined, which are smaller compared to those for operation in the forward active region, in the case of a p⁺np transistor

The Collector's injection efficiency $\gamma_R = \frac{I_{Cp}}{I_{Cp}+I_{Cn}}$ is much smaller than the Emitter's injection efficiency $\gamma = \frac{I_{Ep}}{I_{Ep}+I_{En}}$

$$V_{EB}<0 \rightarrow e^{qV_{EB}/kT} \ll 1$$

$$V_{CB}>0 \rightarrow e^{qV_{CB}/kT} \gg 1$$

$$I_C \cong -qAn_i^2\left[\overset{\sim I_{Cn}\downarrow}{\frac{D_C}{L_C N_C}} + \overset{\sim I_{Cp}\downarrow}{\frac{D_B}{W N_B}}\right] e^{qV_{CB}/k}$$

$$I_E \simeq -qAn_i^2\left[\frac{D_B}{W N_B}\right] e^{qV_{CB}/kT}$$

$$I_B \simeq qAn_i^2\left[\frac{D_C}{L_C N_C}\right] e^{qV_{CB}/kT}$$

All I_C, I_E and $I_B \sim e^{qV_{CB}/kT}$

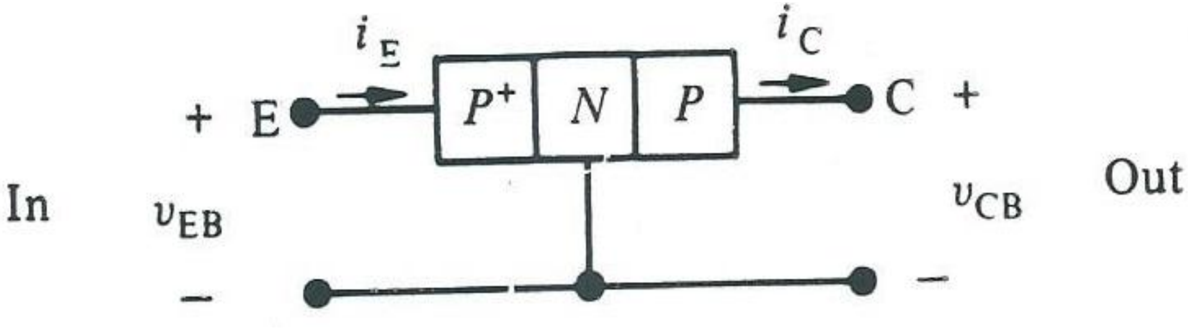
All current components $I_{Cp}, I_{Cn}, I_{Ep}, I_{En}, I_{B1}, I_{B3} < 0$

$I_C < 0$ $I_E = I_{Cp} < 0$ $I_B > 0$

$I_B = I_{B1} - I_{B3} > 0$ since $|I_{B3}| > |I_{B1}|$, due to the electrons injected from Base to Collector

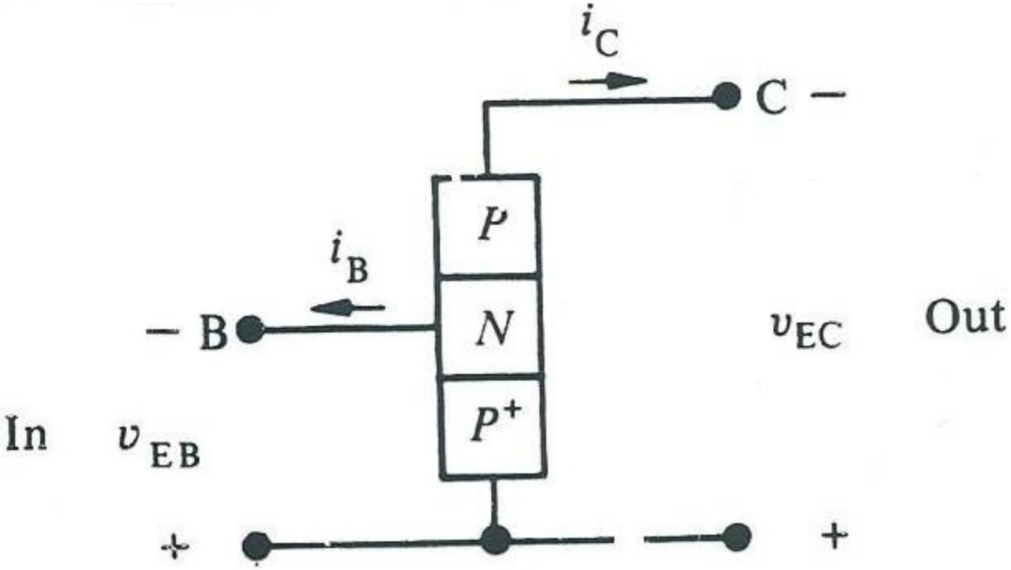
Common Base and Common Emitter connections

Common Base



Input: i_B, v_{EB}
Output: i_C, v_{EC}

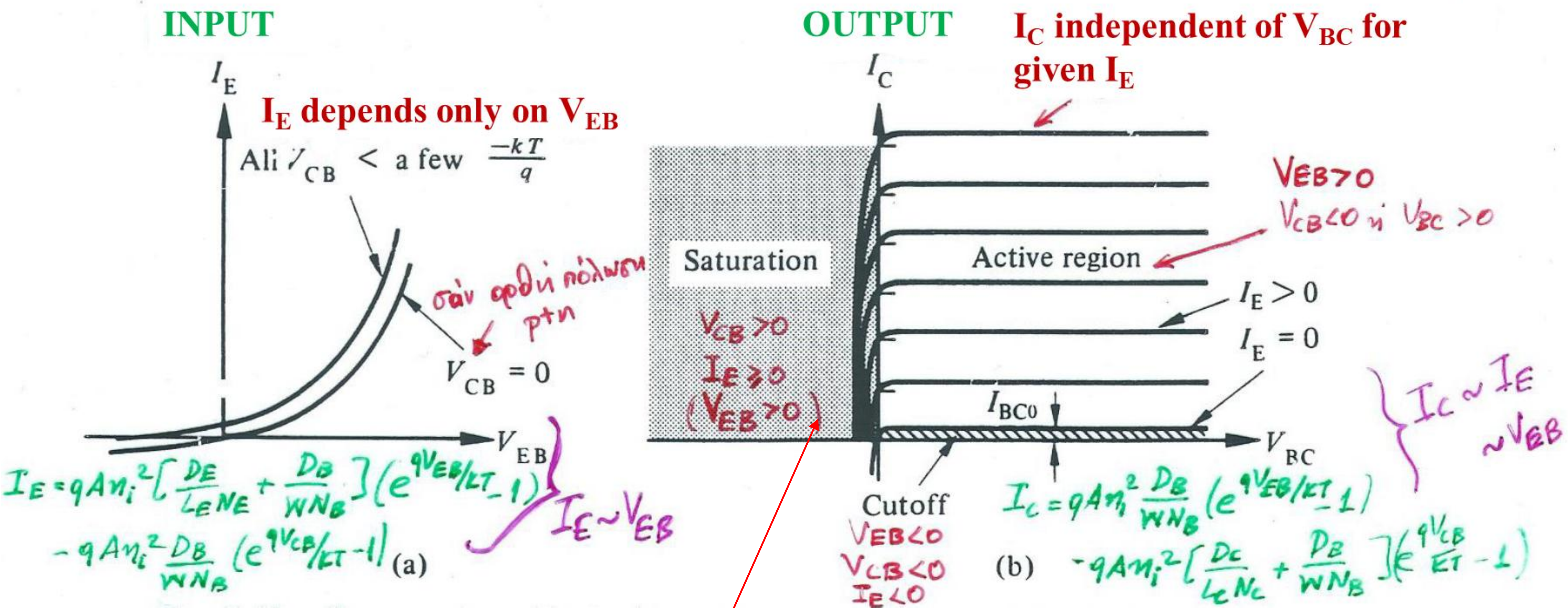
Common Emitter



Input: i_E, v_{EB}
Output: i_C, v_{CB}

Figures from "The Bipolar Junction Transistor", G. W. Neudeck (Addison-Wesley)

Input and Output Ideal I-V for Common Base



Figures from "The Bipolar Junction Transistor", G. W. Neudeck (Addison-Wesley)

For the saturation region:

$V_{CB} > 0$ → C-B forward biased → Injection of holes from C to B

$I_E \geq 0$ ($V_{EB} > 0$) → E-B forward biased → injection of holes from E to B

⇒ **Opposite hole currents at the Collector and I_C decreases with increasing V_{CB}**

Ideal BJT in Common Base – Parameters for active region

Emitter Injection Efficiency **:** $\gamma = \frac{I_{Ep}}{I_{Ep}+I_{En}} \Rightarrow$

$$\gamma_{pnp} = \frac{\frac{D_B p_{B0}}{W}}{\frac{D_B p_{B0}}{W} + \frac{D_E n_{E0}}{L_E}}$$

p⁺n E-B junction $\rightarrow n_{E0} << p_{B0} \Rightarrow \gamma \rightarrow 1.0$

Base Transport Factor **:** $\alpha_T = \frac{I_{Cp}}{I_{Ep}} = 1$ **due to assumption of no recombination in bulk Base region**

Dc alpha **:** $\alpha_{dc} = \frac{I_C}{I_E} \cong \gamma\alpha_T \Rightarrow \alpha_{dc} = \gamma$

$\alpha_{dc} = I_C/I_E$ is the common base short circuit current gain in active region ($V_{CB} = 0$, output's short circuit)

$$\alpha_{dc} = \frac{\frac{D_B p_{B0}}{W}}{\frac{D_B p_{B0}}{W} + \frac{D_E n_{E0}}{L_E}} = \frac{1}{1 + \frac{D_E n_{E0} W}{D_B p_{B0} L_E}} = \left[\frac{1}{1 + \frac{D_E N_B W}{D_B N_E L_E}} = \alpha_{dc} \right]$$

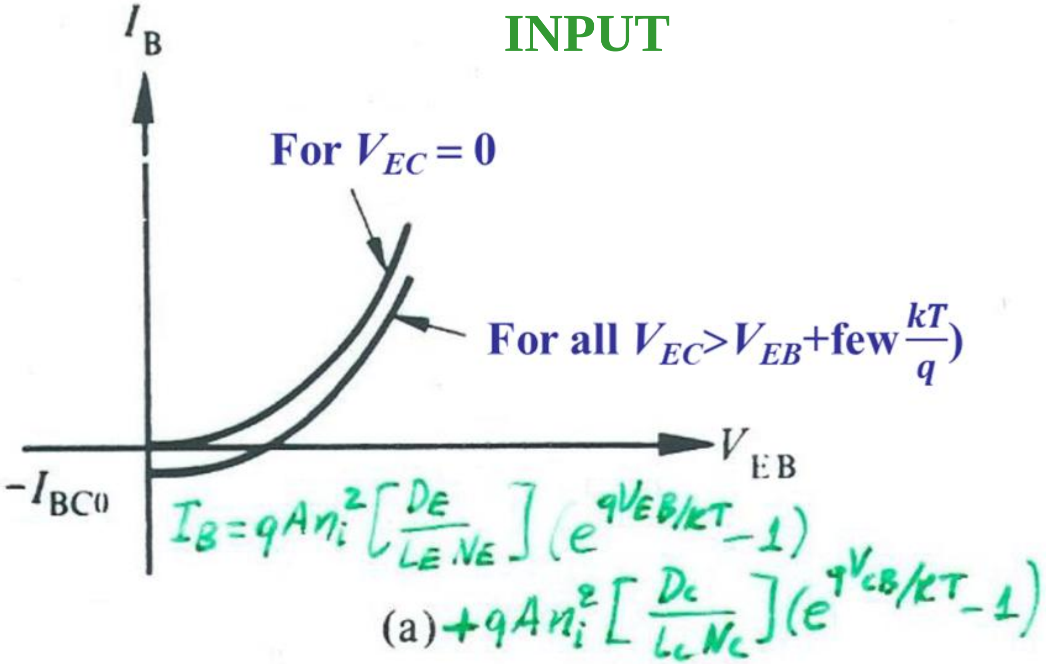
It should be $N_E >> N_B$ and $W << L_E$ for $\alpha_{dc} \rightarrow 1$

In the output I-V, I_{BCO} appears as the collector current I_C for emitter open circuited ($I_E = 0$) and C-B junction reverse biased

Input and Output Ideal I-V for Common Emitter

$$V_{EC} = V_{EB} + V_{BC} \Rightarrow V_{CB} = V_{EB} - V_{EC}$$

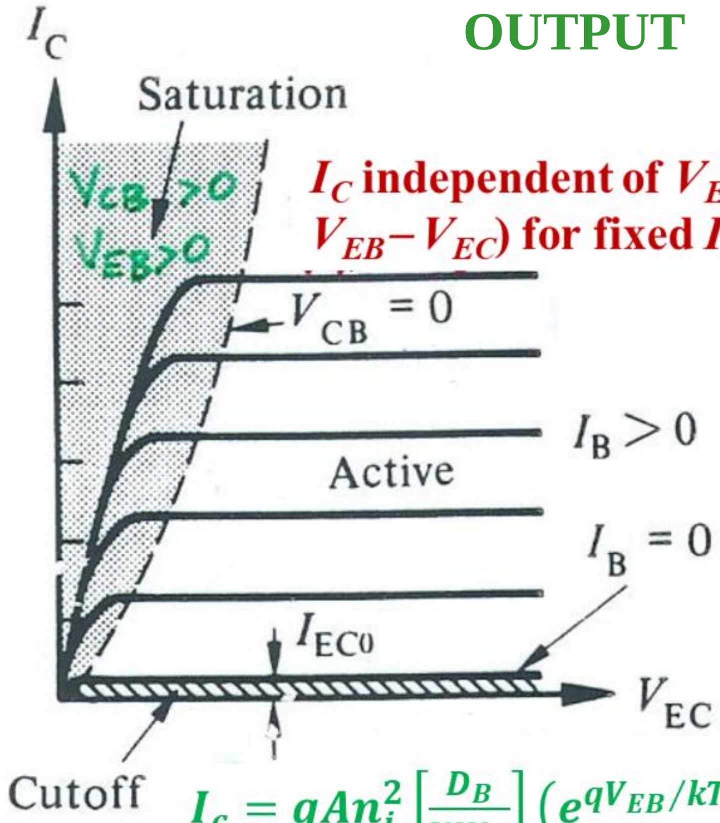
INPUT



$$I_B = qAn_i^2\left[\frac{D_E}{L_E N_E}\right](e^{qV_{EB}/kT} - 1) + qAn_i^2\left[\frac{D_C}{L_C N_C}\right](e^{qV_{CB}/kT} - 1)$$

Figures from "The Bipolar Junction Transistor", G. W. Neudeck (Addison-Wesley)

OUTPUT



I_C independent of V_{EC} (and $V_{CB} = V_{EB} - V_{EC}$) for fixed I_B → fixed V_{EB}

Increase of V_{EC} → more negative V_{CB}

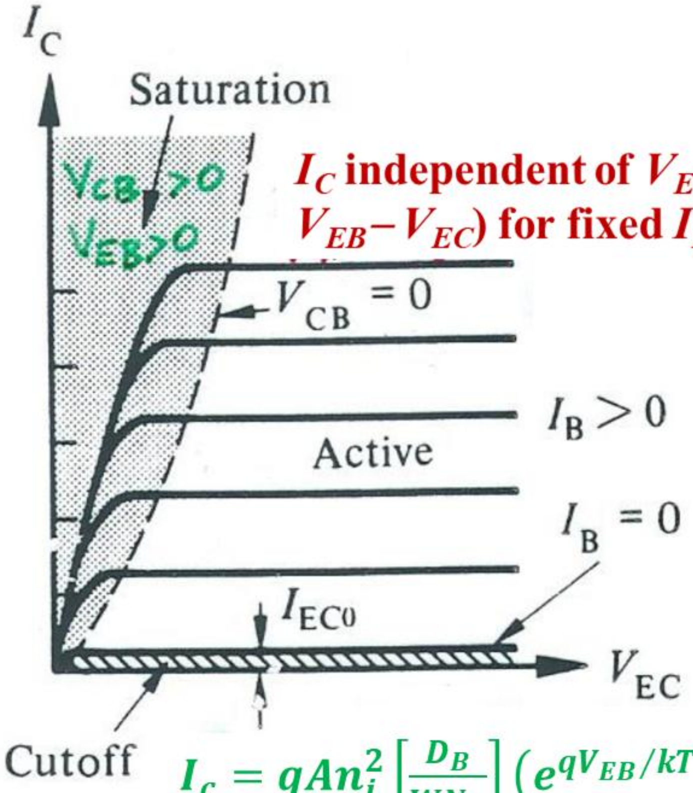
$$I_c = qAn_i^2\left[\frac{D_B}{WN_B}\right]\left(e^{qV_{EB}/kT} - 1\right) - qAn_i^2\left[\frac{D_C}{L_C N_C} + \frac{D_B}{WN_B}\right]\left(e^{qV_{CB}/kT} - 1\right)$$

$$I_B = qAn_i^2\left[\frac{D_E}{L_E N_E} + \frac{D_C}{L_C N_C}\right]\left(e^{qV_{EB}/kT} - 1\right)$$

(i) $V_{EC} = 0 \Rightarrow V_{CB} = V_{EB} \rightarrow$ a diode's I-V

(ii) $V_{EC} > V_{EB} + \frac{3kT}{q} \Rightarrow V_{CB} = V_{EB} - V_{EC} << 0$ reverse bias at C-B. The increase of $|V_{CB}|$ does not affect the injection of electrons from the Base to the Emitter

Output I-V characteristics – Common Emitter



I_C independent of V_{EC} (and $V_{CB}= V_{EB}-V_{EC}$) for fixed $I_B \rightarrow$ fixed V_{EB}

$$I_c = qAn_i^2\left[\frac{D_B}{WN_B}\right]\left(e^{qV_{EB}/kT}-1\right)$$
$$-qAn_i^2\left[\frac{D_C}{L_CN_C}+\frac{D_B}{WN_B}\right]\left(e^{qV_{CB}/kT}-1\right)$$

From "The Bipolar Junction Transistor", G. W. Neudeck (Addison-Wesley)

Active region:

$$\beta_{dc} = \frac{I_C}{I_B} \cong \frac{qAp_{B0}\frac{D_B}{W}e^{qV_{EB}/kT}}{qAn_{E0}\frac{D_E}{L_E}e^{qV_{EB}/kT}}$$ for p$^+$n E-B junction

$$\Rightarrow \beta_{dc} \cong \frac{D_B p_{B0}\, L_E}{D_E n_{E0}\, W} = \frac{D_B N_E\, L_E}{D_E N_B\, W}$$ Current Gain for Common-Emitter

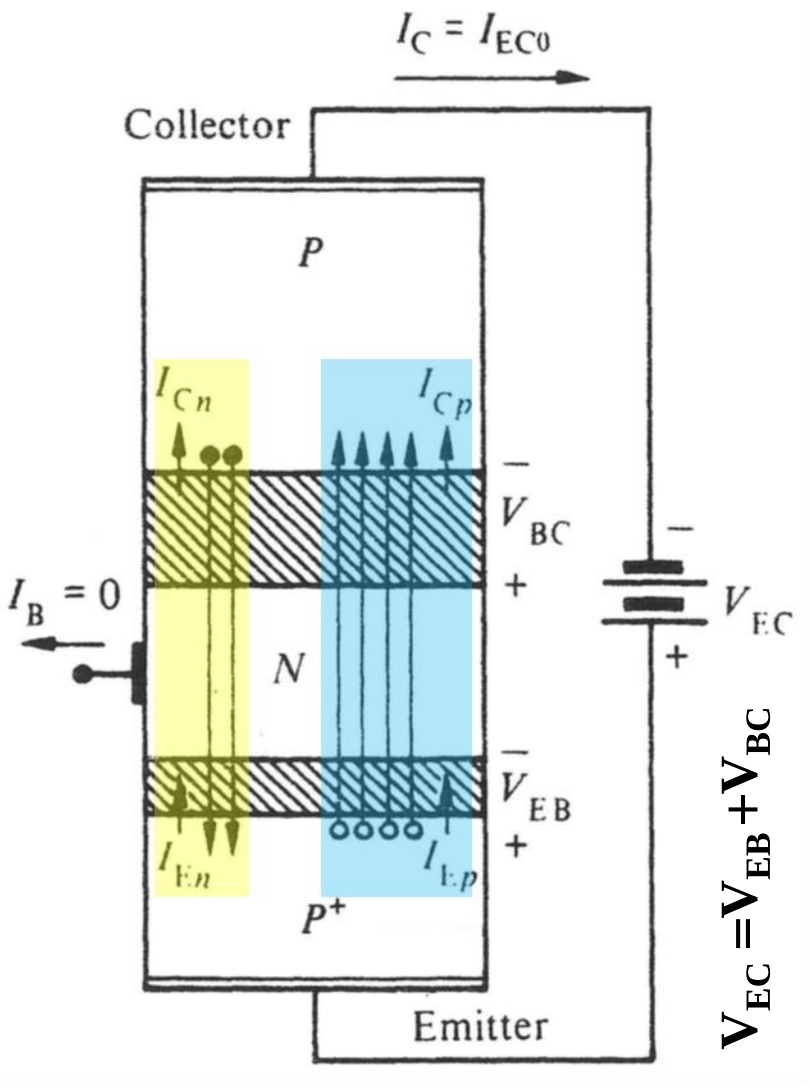
Saturation region: $V_{CB} > 0$ and $V_{BE} > 0 \rightarrow$ forward biased E-B and C-B junctions

In an I_C-V_{EC} curve for constant I_B, the decrease of V_{EC} that reduces I_C corresponds to an increase of V_{CB} and hole injection from C to B and also to a decrease of V_{EB} and hole injection from E to B and C, since

$$I_B = qAn_i^2\left[\frac{D_E}{L_EN_E}\right]\left(e^{qV_{EB}/kT}-1\right) + qAn_i^2\left[\frac{D_C}{L_CN_C}\right]\left(e^{qV_{CB}/kT}-1\right)$$

For I_B constant $\quad V_{EB}\downarrow \quad$ *when* $V_{CB}\uparrow$

The current I_{ECO} for Common Emitter



From "The Bipolar Junction Transistor", G. W. Neudeck (Addison-Wesley)

In the output I-V characteristics, I_{ECO} appears as the collector current I_C for Base open circuited ($I_B = 0$). It is the minimum I_C at the boundary between Active ($I_B > 0$) and Cutoff regions ($I_B < 0$)

It is reported as I_{CEO} in transistor data sheets, which is the actual current direction in npn BJTs

$I_{EC0} = I_{BC0}(\beta_{dc} + 1)$ due the BJT transistor action:

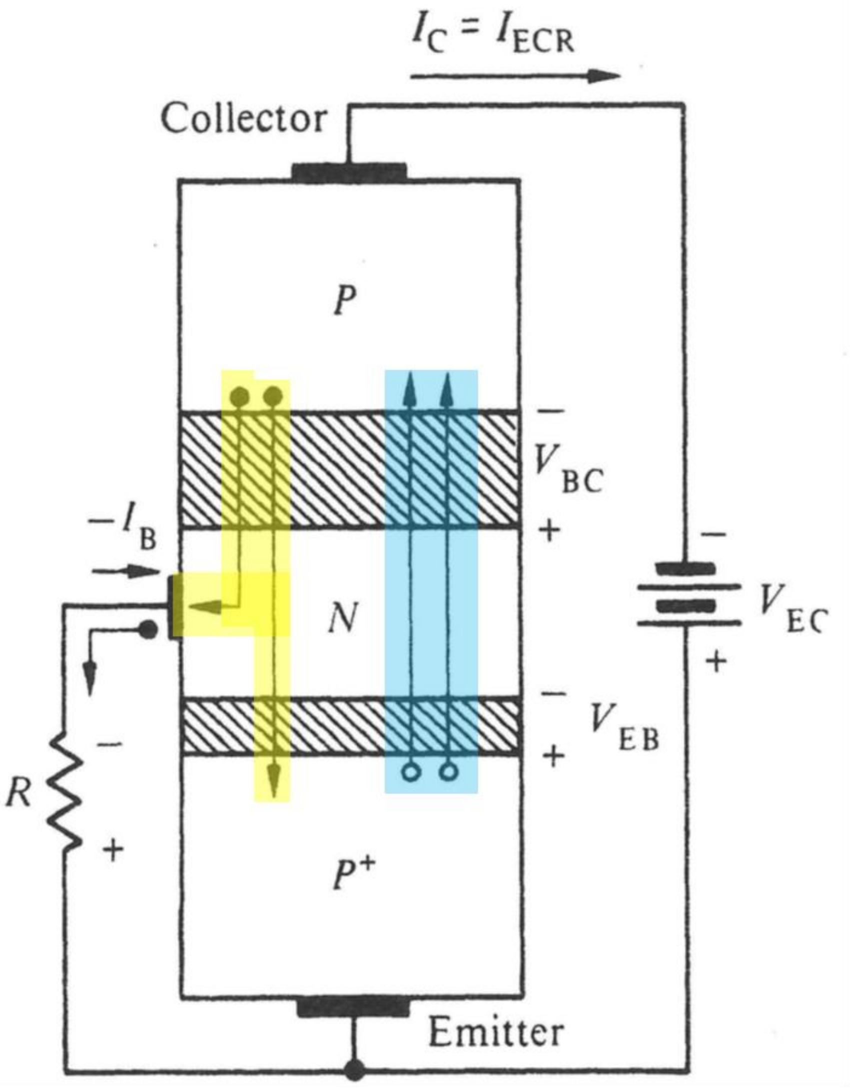
Minority electrons from C drift to B ($V_{CB} < 0$) and, at the B-E junction ($V_{EB} > 0$) they are injected into E: $I_{En} \cong I_{Cn} = I_{BC0}$ (1)

I_{En} trigers a large I_{Ep} of holes injected from E to B, due to $\gamma \to 1.0$. They reach the C-B junction and drift to C: $I_{Cp} \cong I_{Ep}$ (2)

$$\beta_{dc} = \frac{I_C}{I_B} \cong \frac{I_{Ep}}{I_{En}} \Rightarrow I_{Ep} \cong \beta_{dc} I_{En} \cong \beta_{dc} I_{Cn} = \beta_{dc} I_{BC0} \quad (3)$$

$$I_{EC0} = I_{Cn} + I_{Cp} \cong I_{BC0} + I_{Ep} \cong I_{BC0}(\beta_{dc} + 1)$$

About I_{ECO} and I_{ECR} and relationships for npn BJTs



From "The Bipolar Junction Transistor",
G. W. Neudeck (Addison-Wesley)

Connecting a resistor *R* between Base and Emitter reduces I_C from I_{ECO} to a smaller current I_{ECR}

- **Some of the electrons, entering the Base from the Collector, flow through the resistor *R* and are not injected from the Base to the Emitter**
- **This reduces the number of holes injected from the Emitter to the Base and then drifted to the Collector**

If $R = 0$ (short-circuited Base and Emitter) and therefore $V_{EB} = 0$, then $I_C \cong I_{Cn} = I_{BC0}$

Relationships for the currents I_{CEO} and I_{CB0} in npn BJTs:

The relationships can be derived analogously to those for the pnp BJT, by interchanging electrons and holes and reversing the directions of the currents and the polarity of the voltages.

Ebers – Moll Model

Ebers-Moll Equations:

$$I_E = I_F - \alpha_R I_R$$

$$I_C = \alpha_F I_F - I_R$$

$$I_B = (1-\alpha_F) I_F + (1-\alpha_R) I_R$$

They are a simplified set of BJT current equations by collecting terms into only few constants

1) Theory equation:

$$I_E = \underbrace{qA\left[\frac{D_E n_{E0}}{L_E} + \frac{D_B P_{B0}}{W}\right]\left(e^{qV_{EB}/kT} - 1\right)} - \underbrace{\left[\frac{qAD_B}{W} P_{B0}\right]\left(e^{qV_{CB}/kT} - 1\right)}$$

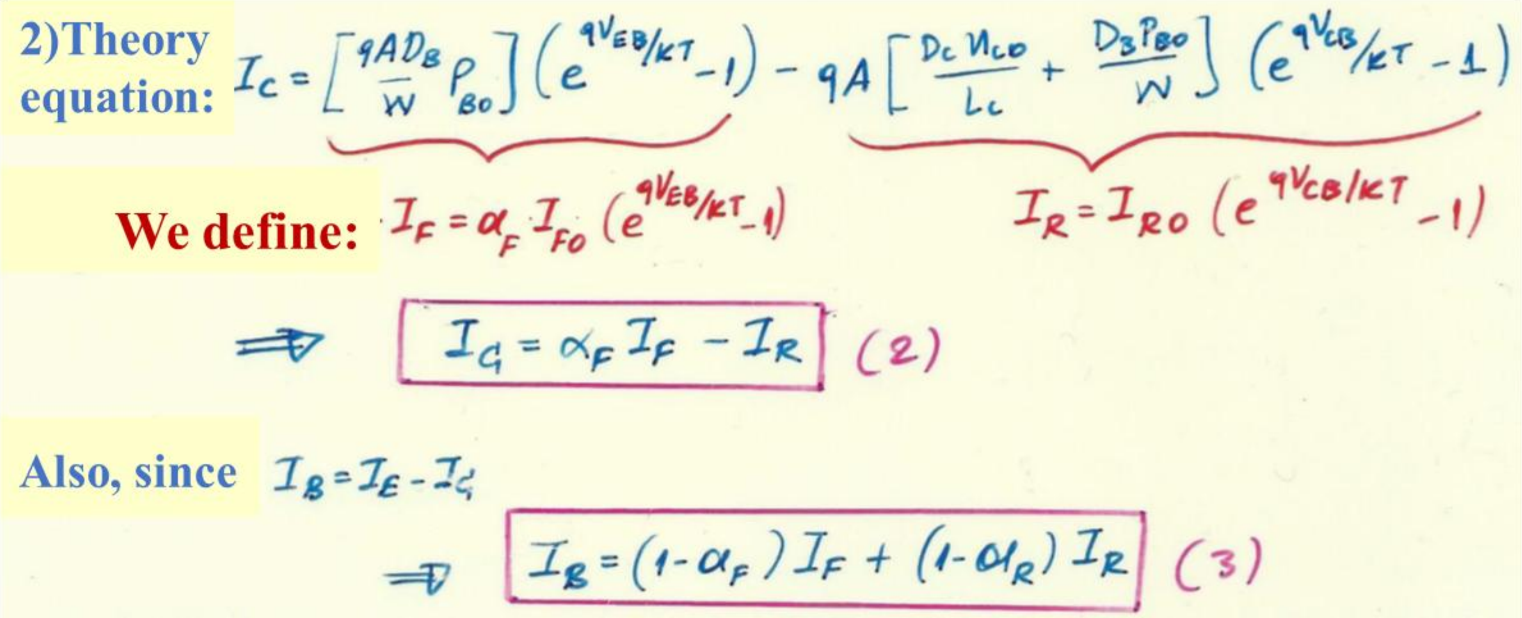
We define: $I_F = I_{F0}\left(e^{qV_{EB}/kT} - 1\right)$ $\alpha_R \cdot I_R = \alpha_R \cdot I_{R0}\left(e^{qV_{CB}/kT} - 1\right)$

where $I_{F0} = qA\left[\frac{D_E n_{E0}}{L_E} + \frac{D_B P_{B0}}{W}\right]$

and $\alpha_R \cdot I_R = \alpha_R \cdot I_{R0}\left(e^{qV_{CB}/kT} - 1\right) = \frac{qAD_B \cdot P_{B0}}{W}\left(e^{qV_{CB}/kT} - 1\right)$

$$\Rightarrow \boxed{I_E = I_F - \alpha_R \cdot I_R} \quad (1)$$

Ebers-Moll equations



2)Theory equation:

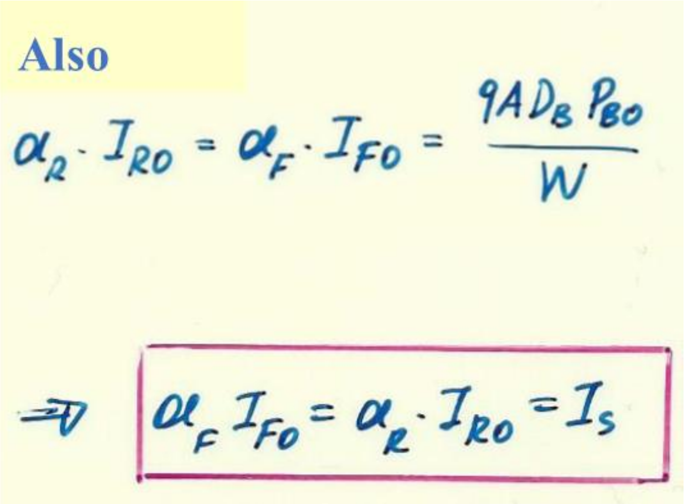
$$I_C = \underbrace{\left[\frac{qAD_B}{W} p_{B0}\right]\left(e^{qV_{EB}/kT}-1\right)} - \underbrace{qA\left[\frac{D_C n_{C0}}{L_C} + \frac{D_B p_{B0}}{W}\right]\left(e^{qV_{CB}/kT}-1\right)}$$

We define: $I_F = \alpha_F I_{F0}\left(e^{qV_{EB}/kT}-1\right)$ $\quad I_R = I_{R0}\left(e^{qV_{CB}/kT}-1\right)$

$$\Rightarrow \boxed{I_C = \alpha_F I_F - I_R} \quad (2)$$

Also, since $I_B = I_E - I_C$

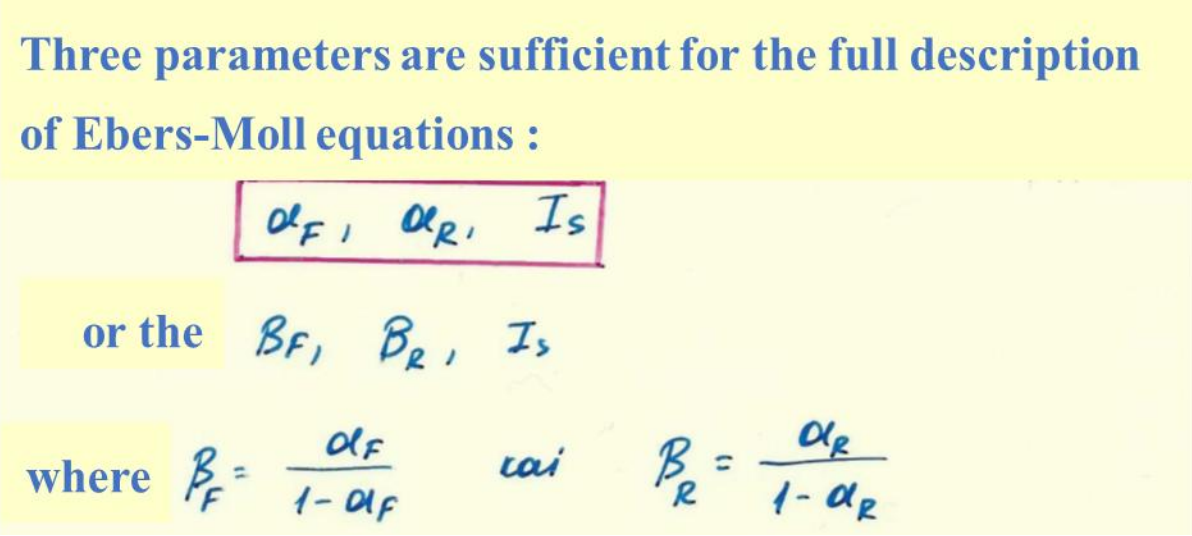
$$\Rightarrow \boxed{I_B = (1-\alpha_F) I_F + (1-\alpha_R) I_R} \quad (3)$$



Also

$$\alpha_R \cdot I_{R0} = \alpha_F \cdot I_{F0} = \frac{qAD_B p_{B0}}{W}$$

$$\Rightarrow \boxed{\alpha_F I_{F0} = \alpha_R \cdot I_{R0} = I_S}$$



Three parameters are sufficient for the full description of Ebers-Moll equations :

$$\boxed{\alpha_F, \quad \alpha_R, \quad I_S}$$

or the $\beta_F, \quad \beta_R, \quad I_s$

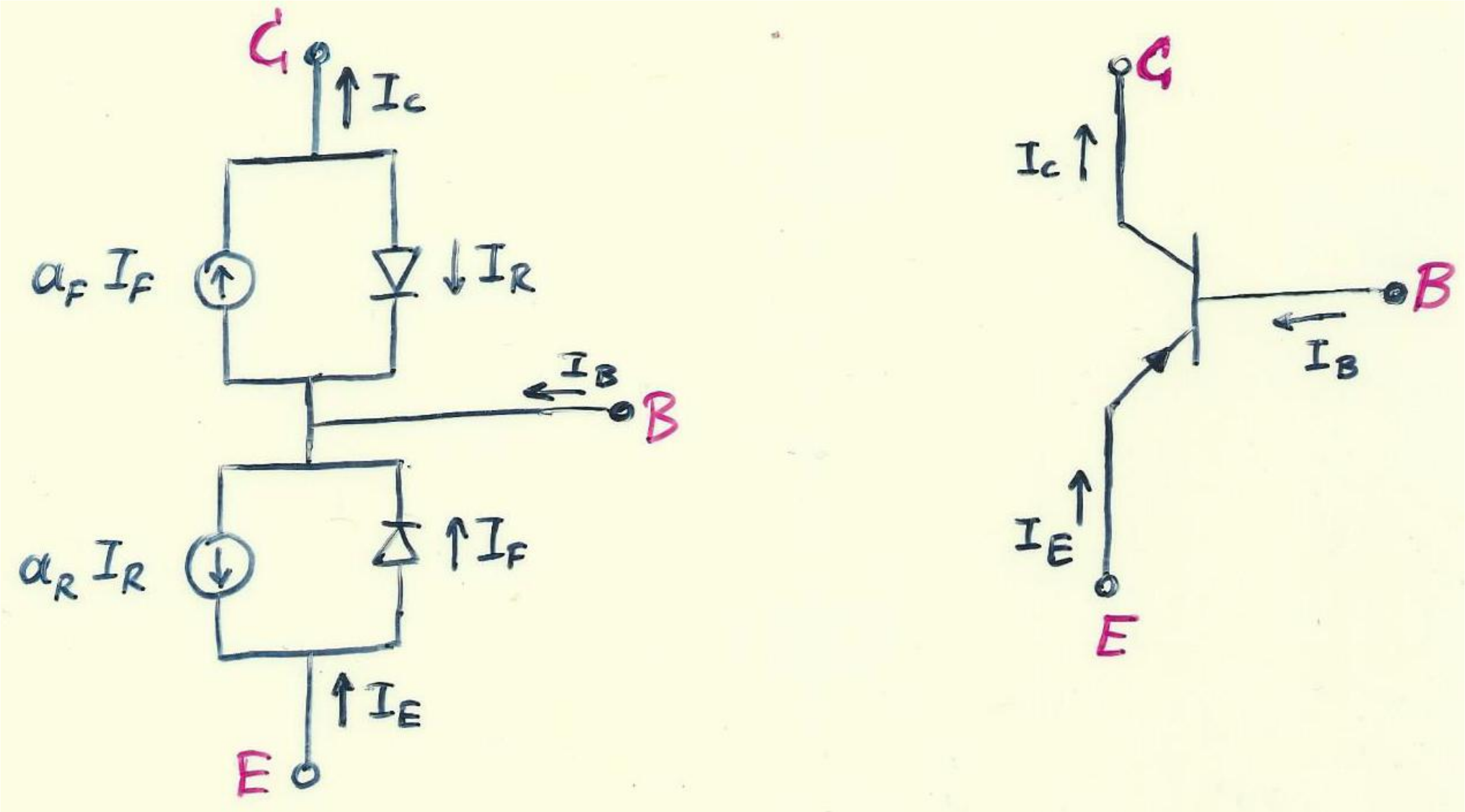
where $\beta_F = \dfrac{\alpha_F}{1-\alpha_F}$ και $\beta_R = \dfrac{\alpha_R}{1-\alpha_R}$

Ebers-Moll equivalent circuit for pnp BJT

$$I_E = I_F - \alpha_R I_R$$

$$I_C = \alpha_F I_F - I_R$$

$$I_B = (1-\alpha_F) I_F + (1-\alpha_R) I_R$$



The Ebers-Moll equations can also be used for non-ideal BJTs, with recombination within the Base, by modifying the I_{R0} and I_{F0} coefficients in front of the exponential terms